Evolutionary Economics and Social Complexity Science

Volume 1

Editors in Chief
Takahiro Fujimoto, Tokyo, Japan
Yuji Aruka, Tokyo, Japan

Editorial Board
Satoshi Sechiyama, Kyoto, Japan
Yoshinori Shiozawa, Osaka, Japan
Kiichiro Yagi, Neyagawa, Japan
Kazuo Yoshida, Kyoto, Japan
Hideaki Aoyama, Kyoto, Japan
Hiroshi Deguchi, Yokohama, Japan
Makoto Nishibe, Sapporo, Japan
Takashi Hashimoto, Nomi, Japan
Masaaki Yoshida, Kawasaki, Japan
Tamotsu Onozaki, Tokyo, Japan
Shu-Heng Chen, Taipei, Taiwan
Dirk Helbing, Zurich, Switzerland

More information about this series at http://www.springer.com/series/11930
The Japanese Association for Evolutionary Economics (JAFEE) always has adhered to its original aim of taking an explicit “integrated” approach. This path has been followed steadfastly since the Association’s establishment in 1997 and, as well, since the inauguration of our international journal in 2004. We have deployed an agenda encompassing a contemporary array of subjects including but not limited to: foundations of institutional and evolutionary economics, criticism of mainstream views in the social sciences, knowledge and learning in socio-economic life, development and innovation of technologies, transformation of industrial organizations and economic systems, experimental studies in economics, agent-based modeling of socio-economic systems, evolution of the governance structure of firms and other organizations, comparison of dynamically changing institutions of the world, and policy proposals in the transformational process of economic life. In short, our starting point is an “integrative science” of evolutionary and institutional views. Furthermore, we always endeavor to stay abreast of newly established methods such as agent-based modeling, socio/econo-physics, and network analysis as part of our integrative links.

More fundamentally, “evolution” in social science is interpreted as an essential key word, i.e., an integrative and/or communicative link to understand and re-domain various preceding dichotomies in the sciences: ontological or epistemological, subjective or objective, homogeneous or heterogeneous, natural or artificial, selfish or altruistic, individualistic or collective, rational or irrational, axiomatic or psychological-based, causal nexus or cyclic networked, optimal or adaptive, microscopic or macroscopic, deterministic or stochastic, historical or theoretical, mathematical or computational, experimental or empirical, agent-based or socio/econo-physical, institutional or evolutionary, regional or global, and so on. The conventional meanings adhering to various traditional dichotomies may be more or less obsolete, to be replaced with more current ones vis-à-vis contemporary academic trends. Thus we are strongly encouraged to integrate some of the conventional dichotomies.

These attempts are not limited to the field of economic sciences, including management sciences, but also include social science in general. In that way, understanding the social profiles of complex science may then be within our reach. In the meantime, contemporary society appears to be evolving into a newly emerging phase, chiefly characterized by an information and communication technology (ICT) mode of production and a service network system replacing the earlier established factory system with a new one that is suited to actual observations. In the face of these changes we are urgently compelled to explore a set of new properties for a new socio/economic system by implementing new ideas. We thus are keen to look for “integrated principles” common to the above-mentioned dichotomies throughout our serial compilation of publications. We are also encouraged to create a new, broader spectrum for establishing a specific method positively integrated in our own original way.
Evolutionary Foundations of Economic Science

How Can Scientists Study Evolving Economic Doctrines from the Last Centuries?
May the worlds be calm and peaceful in the future.

Mozi said: He who rules a large state does not attack small states: he who rules a large house does not molest small houses. The strong does not plunder the weak. The honoured does not demean the humble. The clever does not deceive the stupid. This is beneficial to Heaven above, beneficial to the spirits in the middle sphere, and beneficial to the people below. Being beneficial to these three it is beneficial to all.

Mozi, Book 7: Will of Heaven I (http://ctext.org/mozi)
Preface

This book aims to explain briefly the essential features of the founding theories of economics and compare them with later theories developed to address inconsistencies in outcomes. The earlier stages of this book are focused on the economic ideas and theories developed mainly between the 1930s and 1950s, because their emergence bred what were effectively new branches of economics. Over time, these economic theories have been gradually updated, but this updating has not necessarily addressed their theoretical difficulties. Roughly speaking, the updates converged towards behavioral science without eliminating the essential problems behind the theories. The idea of bounded rationality was a typical concern of these revisions. With universal rationality, then the core of the theory remained. The ideas of systems science were therefore increasingly less associated with this revisionist economic theory. However, even as these updates were being proposed, the world was dramatically changing. To use my favorite phrase, a car is no longer a car, but an adaptive cruising system, an air fighter is no longer an air fighter in the sense that stability is no longer part of its structural design. The control of modern vehicles is becoming further removed from human input. This also applies to the market. The revisionist approach therefore does not fully describe the essential transformations emerging in the world.

For these reasons, I have preferred in this book to describe an alternative analytical framework in the interdisciplinary field of socio-econophysics and socio-dynamics. It targets a set of branching or critical points separating the previous framework from the new one. Arthur (2009) used the term “re-domaining” when he referred to technological innovations. Here, we are trying to re-domain economic theories to fit a new social system. Major technological innovations not only accompany economic and market changes but also alter their meaning. In particular, the evolution of trading technology has changed the meaning of the phrase “invisible hand”. At the end of the last century, the advent of socio-econophysics was decisive in the emergence of a new economic science. This coincided with a change in the economy and the market, which begged a re-domaining of economic science. For
the future, many scientists outside traditional economics are now joining together to
develop new ideas such as power law distribution and network analysis. However,
the more diverse the backgrounds of economic scientists, the fewer common views
they will share, potentially expanding economic terminologies. This book may help
to mitigate any conflicts.

To achieve this, I believed that it was important to position and select the classical
and essential topics in economic science. The behavioral interpretations in the
standard approach rather twisted the economic view to fit a very limited range of
interests. In any retrospection of classical doctrines, even a short one, the ideas of
production, consumption, distribution, and the market must be included. Without
these, no discussion of economic theory is possible. Unfortunately, in the 1980s,
such a synthesis suddenly disappeared from standard economics teaching, so I am
attempting here to resurrect this view. I am convinced that economic science needs
the theory of production and consumption as its first stage.

Incidentally, at this point it may be appropriate to explain my personal views
on ‘economics’. In Japanese, the word ‘economics’ retains an explicit meaning
of a core of morality. In ancient China, there was some metaphysical thought that
underestimated personal business activities because of morality. According to Yao
and Shun, for instance: “If a sage were content simply with eating plain food and
drinking water, then it would make no sense to perform acts of charity for the
people”. In Japan, however, the meaning of economics and business activities was
connected to the *Analects of Confucius* for many years, which eventually led to a
very specific construction of business morality. The most influential mover in this
was Miura Baien, a doctor of Chinese medicine, who systematically represented
his economic doctrine in his book *Kagen* at the end of the eighteenth century.
Interestingly, he not only argued his norm of economic morality but also was
well known as a scholar who successfully formulated the idea of excess demand
to establish price equilibrium.1 In his view, economic policy measures should be
concentrated on the theory of enlightened rule and succor of the people. Profits
must not be monopolized, but should be socially distributed among the people
by the earners themselves, not by either central or local government. Here the
accumulation of monetary wealth became simply a tool to achieve public welfare.
This idea was then applied to capitalism by Viscount Eiichi Shibusawa (1840–1931)
during Japan’s industrial revolution, also known as the Meiji Restoration. Shibusawa
examined an inquiry by Zigong, a disciple of Confucius, who asked “Providing
charity for people widely and thereby relieving them—should that be considered an
act of benevolence?” The master replied that “acts of charity and relief for others
are the works of all ages, or at least should be undertaken by a ruler who governs
a country with justice and benevolence”. In his conscious comparison with Adam

---

Smith, therefore, Shibusawa derived from this Confucian answer that the leadership of a country cannot afford to overlook the importance of industry and profit-making. Summing up, he called his synthesis “harmony between ethics and economy”. In this book, I suggest interpreting Shibusawa’s idea as a coordination between Homo socialis and Homo economicus, according to Dirk Helbing. This is an advanced insight, and the reason why Shibusawa is called “a father of Japanese capitalism”. From the beginning of Japanese capitalism, then, an effort was made to integrate Homo socialis and Homo economicus side by side to manage capitalism. This is the reason why this book does not adopt a one-sided view on Homo economicus alone.

I should also refer to the direct motivation behind this book. The idea of it was born when I was staying at ETH Zürich, i.e., Eidgenössische Technische Hochschule Zürich, recently. I was invited to the institute SOMS, i.e., Sociology, in particular of Modeling and Simulation, by Dirk Helbing in February 2010, August 2011, and March 2012, and also visited the department D-MTEC, i.e., Department of Management, Technology, and Economics, at the instigation of Frank Schweitzer in February to March 2012. Without useful discussions with Dirk and Frank, I would never have written this book, so I am grateful to them both for their hospitality. Needless to say, I am also indebted to many other colleagues, too many to name here, but who will be thanked in specific chapters.

I spent much space on dedications to my professors and colleagues in my previous book (Aruka 2011). Here I would also like to express my gratitude to two other organizations. First, the Japan Association for Evolutionary Economics (JAFEE), where I am currently vice president, and where I continue to be encouraged to study and develop my ideas. Second, and equally, I continue to be enlightened by the Society for Economic Science with Heterogeneous Interacting Agents (ESHIA), where I am a directorial board member. I would like to express my gratitude to all my colleagues at both organizations.

At Springer, I am much indebted to my editors, specifically, Editorial director in charge of Business, Economics, and Statistics, New York, and Editors of Springer Japan. They were kind enough to arrange not only my own book but also the monograph series titled Evolutionary Economics and Social Complexity Science. Without their assistance, this book would not have been published. In particular, Editorial director, New York, decided on the book title: Evolutionary Foundation of Economic Science. His decision is a genuine honor for me.

At my Hakoneyama-Hermitage, Kanagawa Prefecture, Japan
August 2013

---


3 See Chap. 1 of this book.

4 This book was almost written up within 2013. But the book then underwent necessary revisions after 2013. Thus a part of the book is supported by JSPS Grant-in-Aid for Scientific Research (B) no. 26282089, which started since April 2014.
## Contents

1 **Historical Reviews Around Evolving Ideas of the Invisible Hand** ...... 1  
1.1 Modern Technology and the Rise of the Service Economy .............. 1  
1.1.1 The Classical Production Scheme .................................. 2  
1.1.2 A New Production Scheme in View of the Dominance of the Service Economy ........................................... 4  
1.2 The Doctrines of Political Economy in the Anglo-Saxon Tradition ................................................................. 7  
1.3 The Constant Measure of Value and a Nave Reasoning of Technology ................................................................. 9  
1.4 Removal of Ordinary Life from the ‘Invisible Hand’ of Lionel Robbins ................................................................. 10  
1.5 The ‘Invisible Hand’ in the Game Theoretic Approach and the Limit of Computability ......................................... 12  
1.6 The Invisible Hand in the Self-Organization of the Market .......... 13  
1.7 Invisible Hands and Market Failures .................................... 14  
1.7.1 The Trade-Off Between the Real Economy and the Financial Economy, and the Differential Effects on Each .............................................. 15  
1.8 Some Myths of Modern Economies .................................... 16  
1.8.1 Is More Liquidity Better? .................................................. 17  
1.8.2 Are Financial Markets Efficient? .................................... 18  
1.8.3 Is Economics an Equilibrium System? ............................ 20  
1.9 Harmony Between *Homo Economicus* and *Homo Socialis* ........ 21  
1.9.1 Dirk Helbing’s View on Harmony in Society ................... 23  
1.10 The Mozi School: Impartial and Heterogeneous Interacting Agents ................................................................. 25  
1.11 Human Interactions: A Macroscopic Microeconomic Feedback Loop ................................................................. 27  
1.11.1 Economics of a Master Equation and Fluctuations ............ 30  
References .............................................................................. 31
# Contents

## 2 The Historic Design of the Demand Law and Its Reconstruction

- 2.1 Some Criticisms of a Utility Function for the Design of Household Demand ........................................ 35
  - 2.1.1 Consumption As a Compromise Between Self-regarding and Other-Regarding Interests .......... 35
  - 2.1.2 The Discrete Choice Model of Different Modes .......... 37
  - 2.1.3 Some Generalizations on Random Terms, Heterogeneities, and Social Interaction .......... 40
  - 2.1.4 Some Essential Differences Between Frames .......... 43
- 2.2 Analytical Examination of the Demand Law ..................... 45
  - 2.2.1 A Makeshift Idea of Compensated Demand and Income .. 45
  - 2.2.2 Design of the Demand Law and a New Form ............. 46
  - 2.2.3 A Numerical Derivation of a Demand Function .......... 48
  - 2.2.4 The Demand Law as Solved by Hildenbrand (1994) ..... 49
- 2.3 Reconstructing Demand Theory .................................. 51
  - 2.3.1 Self-organizing Patterns of Consumption .......... 52
  - 2.3.2 An Empirical Analysis of Patterns of Consumption ..... 55
- 2.4 The Results of Statistical Verification .......................... 57
  - 2.4.1 The Obtained Distributions of Eigenvalues .......... 57
  - 2.4.2 Comparison Between Alternative Seasonal Adjustments .. 58
- 2.5 Some Implications Derived from Statistical Tests ............. 61
  - 2.5.1 Main Findings ............................................. 61
  - 2.5.2 Further Findings .......................................... 62

## References .................................................................. 63

## 3 Network Analysis of Production and Its Renewal

- 3.1 Changes in the Concept of Price over the Last Century ........ 65
  - 3.1.1 Shift in Trading Methods and the Environmental Niche ........................................ 66
  - 3.1.2 Classical Steps Towards Equilibrium ..................... 66
  - 3.1.3 Application of a Genetic Algorithm to the Economic System ............................................. 67
  - 3.1.4 Significance of the Standard Commodity, in the Context of the Genetic Algorithm .......... 68
- 3.2 The Historical Background to Network Thinking in Economic Theory ............................................. 74
  - 3.2.1 The Recycling of Production .............................. 75
  - 3.2.2 The von Neumann Economy .............................. 77
  - 3.2.3 Von Neumann’s Original Formulation .................. 77
  - 3.2.4 Classical Truncation Rules of Choice of Techniques .... 78
  - 3.2.5 Adaptive Plans in the von Neumann Economy .......... 78
  - 3.2.6 The Market Mechanism as a Genetic Algorithm ........ 79
  - 3.2.7 A System’s Eigenvector to Measure the Profitability of the Fictitious Processes/Commodities .... 80
  - 3.2.8 Minimum Spanning Trees of the Industrial Network .... 80

## References .................................................................. 63
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>An Essential Characteristic of the Joint-Production System</td>
<td>81</td>
</tr>
<tr>
<td>3.3.1</td>
<td>An Acyclic Network of Production</td>
<td>81</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Criticism of the Traditional Approach</td>
<td>85</td>
</tr>
<tr>
<td>3.4</td>
<td>An Empirical Study Using Input–Output Tables</td>
<td>88</td>
</tr>
<tr>
<td>3.4.1</td>
<td>The First Step Towards Empirical Input–Output Analysis</td>
<td>88</td>
</tr>
<tr>
<td>3.4.2</td>
<td>A Further Consideration for Empirical Studies of the Inter-Industrial Network</td>
<td>95</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td>4</td>
<td>Matching Mechanism Differences Between Classical and Financial Markets</td>
<td>101</td>
</tr>
<tr>
<td>4.1</td>
<td>Reconsidering the Law of Supply and Demand in the Free Market</td>
<td>102</td>
</tr>
<tr>
<td>4.1.1</td>
<td>The Classical Auction with Complete Ignorance of Others’ Preferences</td>
<td>102</td>
</tr>
<tr>
<td>4.1.2</td>
<td>Auctions in the Financial Market</td>
<td>105</td>
</tr>
<tr>
<td>4.2</td>
<td>The U-Mart System and Historical Background</td>
<td>111</td>
</tr>
<tr>
<td>4.2.1</td>
<td>The U-Mart System Approach to the Futures Stock Market</td>
<td>111</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Historical Background to the Tokyo Stock Market</td>
<td>112</td>
</tr>
<tr>
<td>4.3</td>
<td>The Matching Mechanisms in the U-Mart Experiment</td>
<td>114</td>
</tr>
<tr>
<td>4.3.1</td>
<td>The Shapes and Performances of the Market Mechanism</td>
<td>114</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Zero-Intelligence Tests in the U-Mart System</td>
<td>116</td>
</tr>
<tr>
<td>4.4</td>
<td>Similarities of Trading Strategies Between SF Spread and Random Strategy</td>
<td>120</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Arbitrage: Equalization Between Markets</td>
<td>120</td>
</tr>
<tr>
<td>4.4.2</td>
<td>The Performance of the Random Agents in U-Mart ver. 4’s Simulation</td>
<td>125</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>130</td>
</tr>
<tr>
<td>5</td>
<td>The Evolution of the Market and Its Growing Complexity</td>
<td>131</td>
</tr>
<tr>
<td>5.1</td>
<td>Practical and Logical Time in High-Frequency Trading (HFT): A Re-domaining of the Trading System</td>
<td>131</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Caveats on HFT from the European Securities and Markets Authority</td>
<td>132</td>
</tr>
<tr>
<td>5.1.2</td>
<td>The Re-domaining of the Market Caused by the HTF System</td>
<td>135</td>
</tr>
<tr>
<td>5.1.3</td>
<td>A Historical Example: A Flash Crash</td>
<td>135</td>
</tr>
<tr>
<td>5.1.4</td>
<td>How a Flash Crash Happened</td>
<td>140</td>
</tr>
<tr>
<td>5.2</td>
<td>A Stealth Market</td>
<td>142</td>
</tr>
<tr>
<td>5.2.1</td>
<td>The Invisible Organization of Finance</td>
<td>142</td>
</tr>
</tbody>
</table>
5.3 Some Instances of Technological Innovations
in the Complex Market Economy ....................................... 145
5.3.1 Redundancies and the Depth of Logic
Contained in a Complex System .................................. 145
5.3.2 Innovation and Techno-Culture .......................... 146
5.3.3 A Creative Coincidence Connected with
Hayabusa’s Return and JAXA’s Evolution .................. 146
5.3.4 An Assessment of the Hayabusa Mission .......... 149
5.4 Key Ideas for the New Economics ......................... 153
5.4.1 The Economics of the Master Equation and Fluctuations.. 153
5.4.2 A General Urn Process ..................................... 155
5.4.3 Pitman’s Chinese Restaurant Process ............ 158
References ..................................................................... 159

6 The Complexities Generated by the Movement of the Market Economy ..................................................... 161
6.1 A Brief Summary of the Efficient Market Hypothesis ........ 161
6.1.1 Disengagements from the Efficient Market Hypothesis... 161
6.2 Moving Away from the Social Philosophy Around
the Gaussian Distribution .............................................. 165
6.2.1 The Historical Penetration of the Gaussian
Distribution and Galton’s Ideas .................................. 165
6.3 Heavy Tail Distributions with Heavier Randomness .... 170
6.3.1 Hazard Rates ..................................................... 172
6.3.2 An Alternative Derivation in View of
Memoryless Processes .............................................. 176
6.3.3 Some Empirical Findings in the Market ............ 177
6.4 Alternative Interpretation: Trader Dynamics to Generate
Financial Complexity ..................................................... 178
6.4.1 Rules to Be Specified ....................................... 181
6.4.2 Complexities in a Dealing Model of an Iterated
Finite Automaton ..................................................... 190
References ..................................................................... 194

A Avatamsaka Stochastic Process ..................................... 195
A.1 Interactions in Traditional Game Theory and Their Problems .... 195
A.1.1 A Two-Person Game of Heterogeneous
Interaction: Avatamsaka Game .......................... 196
A.1.2 Dilemmas Geometrically Depicted: Tanimoto’s
Diagram ............................................................. 197
A.2 Avatamsaka Stochastic Process Under a Given Payoff Matrix .. 199
A.2.1 Avatamsaka Stochastic Process Under Various
Payoff Matrices ..................................................... 200
B The JAVA Program of URandom Strategy ........................................ 203

C An Elementary Derivation of the One-Dimensional Central
Limit Theorem from the Random Walk ........................................ 207
  C.1 The Derivation of the Density Function of the Normal
      Distribution ........................................................................ 209
  C.2 A Heuristic Finding in the Random Walk ............................. 213
      References ........................................................................... 213

Name Index .................................................................................. 215

Subject Index ............................................................................... 217
### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>AI</td>
<td>Artificial Intelligence</td>
</tr>
<tr>
<td>AR</td>
<td>AutoRegressive</td>
</tr>
<tr>
<td>ARIMA</td>
<td>AutoRegressive Integrated Moving Average</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CLT</td>
<td>Central Limit Theorem</td>
</tr>
<tr>
<td>DFR</td>
<td>Decreasing Failure Rate</td>
</tr>
<tr>
<td>DNA</td>
<td>DeoxyriboNucleic Acid</td>
</tr>
<tr>
<td>EMH</td>
<td>Efficient Market Hypothesis</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>GCLT</td>
<td>Generalized Central Limit Theorem</td>
</tr>
<tr>
<td>HFT</td>
<td>High Frequency Trade</td>
</tr>
<tr>
<td>ICT</td>
<td>Information and communication Technology</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>independent, identically distributed</td>
</tr>
<tr>
<td>IIA</td>
<td>Independence from Irrelevant Alternatives</td>
</tr>
<tr>
<td>JAXA</td>
<td>Japan Aerospace Exploration Agency</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Programming</td>
</tr>
<tr>
<td>LTCM</td>
<td>Long-Term Capital Management</td>
</tr>
<tr>
<td>MA</td>
<td>Moving Average</td>
</tr>
<tr>
<td>MNL</td>
<td>Multinomial Logit</td>
</tr>
<tr>
<td>MST</td>
<td>Minimum Spanning Tree</td>
</tr>
<tr>
<td>NFA</td>
<td>Net Foreign Assets</td>
</tr>
<tr>
<td>OR</td>
<td>Operations Research</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>RMT</td>
<td>Random Matrix Theory</td>
</tr>
<tr>
<td>SBS</td>
<td>Shadow Banking System</td>
</tr>
<tr>
<td>TSE</td>
<td>Tokyo Stock Exchange</td>
</tr>
</tbody>
</table>
Chapter 1
Historical Reviews Around Evolving Ideas of the Invisible Hand

Abstract This book aims to identify several points at which new economic theories diverged from the old by providing an overview of the traditional theories. The market is now dominated by the influence of financial speculation. That is to say, speculation has long since passed its original function. We can no longer remain in the domain of traditional definitions around markets. So our retrospection of traditional economics is limited to special interests. We do, however, give a quick characterization of the economy first, and we will then discuss historical ideas.

1.1 Modern Technology and the Rise of the Service Economy

To clarify the evolution of modern economics, I will first discuss modern technology. In the past, it was normal for industrial processes to improve a piece of technology to make it as efficient as possible, and then manage it for a longer period, since it took quite a long time to process a basic principle into a usable machine. Modern technology, however, is likely to have moved much more quickly from basic science to usable technology. So-called nanotechnology is a good example of this. In other words, it now takes much more time to generate a new basic principle than a new engineering process. In general, basic science provides us with a huge number of possibilities, which can lead to potentially many more combinations of technologies. Therefore, detecting a feasible technology will be a much more difficult task. This may require better techniques underlying statistical physics/mechanics. It may also apply to Information and Communication Technology (ICT).

ICT ensures that one of the core engines of society, particularly for services management, is also common to daily life for end-users as well as in production. Specifically, ICT can intermediate activities, and then provide ‘services’ bilaterally. The ways in which ICT is connected to societal activities and service processes must be nonlinear, dynamic, and hierarchical over time, i.e., complex and/or synergetic. ICT is therefore naturally an attractive research subject for complex sciences. At the same time, future society is likely to be built on ICT intermediation.

It is therefore obvious that ICT will be decisive in our future as both hardware and software. In future, ICT may be a key factor, not only crucial to each process of production at the various different stages of an economic system, but also in
generating new technologies. Looking at ICT, then, it is possible to generate a bird’s-eye view of the future profile of society. In this sense, designing ICT will serve as a policy measure to assess a social activity. In short, the new service-dominant economy will be realized if, and only if, ICT progresses as expected.

During the final quarter of the last century, the economy shifted towards services and away from manufacturing. This transition is part of the ongoing evolution of the modern economy since the industrial revolution, and in particular, changes in production and consumption. More detailed observations on this will be found in Chaps. 2–3. Until the advent of the dominance of the service economy,¹ the basic principles of production and consumption, either in classic political economy or neoclassical economics, depended on the view that the consumption of citizens should be achieved outside the economic process. In the neoclassical view, the household will by definition never participate in any productive process except by providing its labor, because the household cannot be a resource for production (productive factor). Even the reproduction of the labor force should be outside the bounds of production, because its negation implies slavery: the household as a labor production factory.

### 1.1.1 The Classical Production Scheme

For future reference, I will summarize here the traditional production scheme. We are used to a time profile of production starting from the initial period, when the primary factor enters the process, through the intermediate product, to the final one. There are two kinds of primary factor: land and labor. “Consumption”, the final column of the table, is outside the economic process. This means a complete separation of production from consumption. The traditional production scheme cannot help but be broken down if there is any intersection between production and consumption. The introduction of service-dominant processes suggests the emergence of such an intersection (Fig. 1.1).

The actual time profile may become much more complicated with the existence of durable capital goods. The diagonal elements on the table show the vertically integrated process in this scheme of production, while the rows show the horizontally cross-operated process. The gray cells \{κ₁(0), κ₂(0), \{t₁(1), \{t₁(2)\}\} show the time structure of production. The bold framework shows the input–output relationship, with the time structure synchronized as a simultaneous system of production of general form with multiple final products, i.e., a joint-product system (Table 1.1):

---

¹ ‘Service-dominant’ is often used to refer to ‘service-dominant technology’, as distinguished from ‘dominance of the service economy’.
Table 1.1 The classical production scheme

<table>
<thead>
<tr>
<th>Time</th>
<th>The primary factor</th>
<th>The intermediate product</th>
<th>The final product</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>The initial period</td>
<td>( k_1(0), k_2(0) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The first period</td>
<td>( k_1(1), k_2(1) )</td>
<td>( t_1(1) )</td>
<td>( \xi_1(1) )</td>
<td>Minus ( \xi_1(1) )</td>
</tr>
<tr>
<td>The second period</td>
<td>( k_1(2), k_2(2) )</td>
<td>( t_1(2) )</td>
<td>( \xi_1(2) )</td>
<td>Minus ( \xi_1(2) )</td>
</tr>
</tbody>
</table>

Table 1.2 The simultaneous system of production

<table>
<thead>
<tr>
<th>Multiple processes</th>
<th>Multiple inputs</th>
<th>Multiple products</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process (1)</td>
<td>( t_1(1), t_2(1), t_3(1) )</td>
<td>( \xi_1(1), \xi_2(1) )</td>
</tr>
<tr>
<td>Process (2)</td>
<td>( t_1(2), t_2(2), t_3(2) )</td>
<td>( \xi_1(2), \xi_2(2) )</td>
</tr>
</tbody>
</table>

Note: This is an extension of Table 10.1 in Aruka (2012, p. 165) from a single durable capital good to multiple durable capital goods.

In Table 1.2, the periods can be interpreted for process 1, 2, and so forth, by dropping the times and establishing the von Neumann balanced growth scheme of production first generated in the 1930s.²

The replacement of land with capital as a primary factor has been promoted because of the prevalence of the Cobb-Douglas-type production function since the 1930s. However, its replacement was not recommended in a scientific sense, because capital is currently used as a reproducible good except in particular circumstances. Capital, after industrialization, must not be primarily perpetuated in a natural sense.

²Further information on the details of von Neumann’s economic growth model can be found in von Neumann (1937), and see, e.g., Aruka and Koyama (2011, pp. 9–16; 2012, pp. 165–170).
Capital is merely an intermediate product even if it can be carried over for more than 1 year. Capital, in the production function, is a real product (and not financial) if its durability is a year, and a virtual intermediate product of a fictitious process if its durability is more than a year. Capital cannot be given the same status as labor in the production function.

1.1.2 A New Production Scheme in View of the Dominance of the Service Economy

This scheme is important in the new century, with the prevalence of the service-dominant economy. This new style of production characterizes the fundamental propositions of the economy. In advanced countries, the ratio of service production to GDP is formally estimated to be over 70 percent on average, but this estimation excludes service activities in manufacturing industries. Taking the service activities of the ICT industry as an example, a major company like IBM often earns over two-thirds of its pre-tax income from service provision, as Fig. 1.2 shows. High productivity in manufacturing may therefore be mainly from service activities, so it is vital that we understand the essential features of service production and the service economy.

The attributes of a service, as distinct from a tangible good, may be defined as follows:

1. The service is intangible
2. Provision and consumption cannot be separated, but must occur at the same time
3. It will have different effects for different people, especially in terms of physical efficiency and mental satisfaction
4. Stock (inventory) is either perishable or cannot be held
5. The customer participates in a value assessment process

It is possible to define service-dominant logic following the work of service scientists.\(^3\) They defined service as “the application of specialized competences (knowledge and skills) through deeds, processes, and performances for the benefit of another entity or the entity itself”.

According to Vargo and Lusch (2004b), there are ten fundamental principles of service-dominant logic in a socioeconomic system:

1. Service is the fundamental basis of exchange.
2. Indirect exchange masks the fundamental basis of exchange.
3. Goods are distribution mechanisms for service production.
4. Operant resources are the fundamental source of competitive advantage.
5. All economies are service economies.
6. The customer is always a co-creator of value.
7. The enterprise cannot deliver value, but only offers value propositions.
8. A service-centered view is inherently customer-oriented and relational.
9. All social and economic actors are resource integrators.
10. Value is always uniquely and phenomenologically determined by the beneficiary.

It seems reasonable to change our view of production to one of service dominance. This new view suggests a *Copernican revolution* of economic theory as an economic science. An attempted change such as the introduction of bounded rationality, which is similar to attempting to introduce epicycles to justify geocentrism, will no longer suffice to rescue the old-fashioned view. Therefore, the way that we produce and consume has changed profoundly, and a structural change is required that accommodates the unpredictability of human behavior.

### 1.1.2.1 The Collapse of Market Independence

Alongside the transition of the economy towards service dominance, we also face the collapse of market independence. This phenomenon may reinforce the econ-

---

\(^3\)According to Professor Kazuyoshi Hidaka, Tokyo Institute of Technology, there were/are a wide variety of the notion of service. In service marketing, IHIP is well known as the description of service properties:

- Intangibility
- Heterogeneity
- Inseparability = Simultaneity
- Perishability

In addition, ‘Value Co-creation’ may be also included. See Lovelock and Gummesson (2004), Vargo and Lusch (2004a), and Laroche et al. (2001).
In traditional economic theory, the market is independent of production and consumption. The supply curves are independently derived from production, which itself is separate from consumption. Likewise, the demand curves are independently derived from consumption (the utility function, for instance). The market is simply defined as the *field for the interaction* between the supply and demand curves, which are mutually distinct, as shown in Fig. 1.3. However, as service dominance develops, this will change, and market independence will break down. Some of the market activities will be internalized into production and consumption, as shown in Fig. 1.4.

I conclude, therefore, that traditional economics has been restrictively limited, especially in terms of its links to actual activities. In Chaps. 4 and 5, I will discuss this transfiguration of market image, but before that, I will examine why and how
traditional economics has been forced ever further from reality. It is but a short distance from the doctrines of political economy to the doctrines of economics.

1.2 The Doctrines of Political Economy in the Anglo-Saxon Tradition

We must lightly touch on the prehistory of economics by considering briefly the doctrines of political economy in the Anglo-Saxon tradition. The word ‘economy’ in medieval times originally referred to the household of the feudal landlord. ‘Political’ meant taxation on citizens and farmers. The language of political economy still remains at prestigious universities. For instance, the title “Drummond Professor of Political Economy” at Oxford University is still given to the holder of a chair in neoclassical economics.

The main subjects of political economy were the theory of value, the law of population, and the theory of international trade and comparative advantage. David Ricardo (1772–1823) was one of the greatest scholars of political economy. Thomas Malthus (1766–1834) was also influential, writing about population dynamics from an economic standpoint. His works greatly influenced biologists, providing the first real interaction between economics and biological sciences. The next link emerged from the invention of game theory by von Neumann and Morgenstern (1944), followed by evolutionary game theory, proposed by Maynard Smith and Price (1973). Evolutionary game theory was the first interaction drawing from biologists to economists rather than the reverse.

David Ricardo’s representative work was *On the Principles of Political Economy and Taxation* (Ricardo 1817). He put forward many arguments, two of which are particularly well known, the differential rent theory and the international trade theory, or theory of comparative advantage.

The differential rent theory states that rent arises because of the heterogeneity of lands, i.e., from differences in the fertility or location of agricultural land. These factors are the result of evolution of technology as well as the transpiration system. In this theory, rent on the worst area becomes zero over time. The boundary condition of rent on the worst land is usually zero and then becomes positive as the margin of cultivation is extended. Land rent may increase as the population increases.

The international trade theory or theory of comparative advantage states that all nations could benefit from free trade through specialization of their industry structure, even if that nation is less efficient at producing all kinds of goods than its

---

4 Maynard Smith has contributed much to the Hawk-Dove game. In 1973 Maynard Smith formulated ‘evolutionary game framework’ as well as the idea of ‘evolutionary stable equilibrium (ESS)’. Also see Maynard Smith (1982), his book to discuss evolution and the Theory of Games.

5 Note that the differential rent theory was first published in 1777 by James Anderson (1739–1808).
trading partners. The theory, however, contains a special imperative that both nations committed to trade must agree to mutual specialization of their economies, to generate higher domestic efficiency. For many years, there were few generalizations of comparative advantage of international trade theory towards a general case of \( n \) commodities and \( m \) countries. It was only in 2007 that a mathematically complete proof of a many-country, many-commodity case with intermediate goods and choice of production techniques was published by Shiozawa (2007).

Both propositions are important to demonstrate a relative effect. Ricardo focused on the differences and the interaction of the qualities of production factors either domestically or internationally. The first proposition is quite ingenious in envisaging that fertilization of land and a change of environment around it, for example because of the building of a railway, can change the quality of the land. The second proposition was criticized by his contemporaries in view of the need for nations to defend their own positions, a criticism inherited from List (1841, 1910) in Austria.

Ricardo’s priority of economic efficiency was also found in the Corn Law debates in the United Kingdom, about a protective policy for agricultural produce. At that time, the price of corn was regarded as the wage rate for workers. When the Corn Laws were abolished in 1846, the real wage rate could increase. Securing this increase required an increase in the importing of cheaper corn from overseas. As a result, the international division of labor was accelerated between the United Kingdom and her partner countries.

The interest in heterogeneity and interaction was not inherited by the neoclassical economists, who accepted only those parts of Ricardo’s theories that dealt with homogeneities of labor and capital around the laws of supply and demand. In the nineteenth century, both Europe and the United States were exposed to a movement to establish factory systems, which was motivated by a new series of technological innovations, and required a homogeneous labor force and capital. Homogeneity is very important for industry. Neoclassical economists naturally accepted this dominant direction of travel.

There were three great fathers of neoclassical economics, Walras (1874), Menger (1871), and Jevons (1871), who set out the Marginal Revolution of Economics in the 1870s. Before them, however, Gossen (1854) formulated the theory of marginal utility in a complete mathematical form. His father was a Prussian tax collector, so he was obliged to pursue that path, abandoning his interest in cosmology, leading to a developing interest in economic theory. The marginal theory of utility requires the assumption of ‘independent individuals’ whose preferences are not distorted by the external world. This assumption may be reminiscent of a statement of inviolable individual right, but also enables an internal comparison between heterogeneous individuals. To secure the design of individualism, heterogeneities were regarded as exceptional.

A similar civilization to that driven by the English industrial revolution also emerged in Prussia. Neoclassical economics accepted all the new ‘ideal’ features of the industrial revolution, and therefore accepted the idea of a classical market with homogeneous agents. Similarly, just as neoclassical economics is a product of its historical origin, we can also understand that our economic theories must change
over time, as the social or institutional setting changes. This understanding may naturally bring new developments in economics.

1.3 The Constant Measure of Value and a Nave Reasoning of Technology

Following Adam Smith, David Ricardo developed the labor theory of value, creating two different theories: the theory of embodied labor and the theory of commanded labor. The former was used by Karl Marx, whose well-known theory of exploitation has now largely been discredited. Ricardo’s theory only holds if there is a limited system where each productive process produces a single product. In other cases, a price system with any positive rate of profit may diverge from a labor value, which means the exact quantity of labor embodied directly and indirectly to produce the goods. In the market, the price of a good may be influenced by distributive factors such as rate of profit or wages. The indirect embodied labor depends on the interactive processes of embodied labor, which can give rise to a nonlinear fluctuation because of its network of production. Ricardo intuitively recognized such an effect, but only followed a nave idea of the division of labor.

Instead of analyzing a network effect, he was looking for a constant measure of value, which would ideally reflect a real quantity of embodied labor. This idea was revived by Piero Sraffa (1898–1983), an Italian economist working at Trinity College, Cambridge, who in 1960 proved the constant measure of value to be uniquely the maximal eigenvector of a feasible productive network with a given rate of profit and wages. Sraffa called such an eigenvector the standard commodity. He also proved his standard commodity could exist in multiple forms in a more general case of a jointly productive system, which is largely the von Neumann balanced growth model. Unlike Ricardo, as Klaus Mainzer, a philosopher of science at TUM, i.e., Technische Universität München, pointed out, the German historical school had already understood the idea of a productive network and its nonlinear effect.

The price system of a standard commodity cannot be solved without matrix theory, which enables the observation of an interactive production process. The idea of networks thus continued to be suppressed in the neoclassical tradition. This was reinforced not only by David Ricardo but also by Alfred Marshall (1842–1924), who navey accepted the idea of division of labor (Marshall 1890). It is this tradition that has generated the world of homogeneity. Marshall (1890, p. 271), quoted by Brian Arthur (2009, p. 160), said:

> [I]f one man starts a new idea, it is taken up by others and combined with suggestions of their own; and thus it becomes the source of further new ideas. And presently subsidiary trades grow up in the neighborhood, supplying it with implements and materials, organizing its traffic, and in many ways conducing to the economy of its material.

---

6His suggestion on the German historical school was made during a conversation with the author at TU Munich in March 2012.
Things have not changed since Marshall’s day. If anything, the mysteries of the trade are now deeper. This is because they are more likely to be grounded in quantum mechanics, or computation, or molecular biology. Formal versions of the craft do find their way eventually into technical papers and textbooks. But the real expertise resides largely where it was created, taken for granted, shared, and unspoken. It follows that once a region, or a country for that matter, gets ahead in an advanced area of technology, it tends to continue to progress further ahead. Success brings success, so that there are positive feedbacks or increasing returns to regional concentrations of technology. Once a small cluster of firms builds up around a new body of technology, it attracts further firms.

A path-dependent process may occur after any creative accident, so government can positively exert its initiative for innovation. Marshall’s view, however, still survives to bring some powerful prejudices to bear on actual innovations.

1.4 Removal of Ordinary Life from the ‘Invisible Hand’ of Lionel Robbins

Alfred Marshall was a great economist, who established the *Tripos for economics* in 1903 at the University of Cambridge, marking the beginning of economics as an established discipline in universities around the world. He also supervised John Maynard Keynes at Cambridge. Marshall, in the Anglo-Saxon tradition, naturally inherited Ricardian principles, but renovated them as neoclassical principles of partial equilibrium. His partial equilibrium analysis, in contrast with general equilibrium, is used to this day. His analysis is made partial by the use of the condition *ceteris paribus*.

There is a decisive difference between classical economics and neoclassical economics. In the latter tradition, Alfred Marshall limited economics to the study of the ordinary business of life (Marshall 1890, Preface 20, I.I.1, I.II.1, I.II.6, I.II.8, I.II.16).

Economics is therefore the study of man in the ordinary business of life. It enquires how he gets his income and how he uses it. It is the study of wealth, and more importantly, it is part of the study of man. It is clear that ordinary life is essential to this inquiry. This idea underlies both the sustainability of ordinary life of members of the economy and the universality of the market. Marshall’s profile of the economic framework was then severely restricted by Lionel Robbins (1898–1984) in 1932, who insisted that economics was a theory of optimization of a limited amount of economic resources. Economics is therefore a science that studies human behavior as a relationship between ends and scarce means that have various alternative uses.

This idea does not always require Marshall’s supposition of the ordinary life. From his restrictive definition, economics deprived of the ordinary life was born. Respectable economic chivalry in ordinary life, Marshall argued, was no longer permissible in economics (Marshall 1890, VI.XIII.68, VI, XIII.74, Note 168, App.A 37). This therefore marks a branching point from classical economics.
Robbins’ doctrine of economics has been a kind of optimizer. He believed that his economic optimizer could be compatible with various settings, which can give rise to a problem identifying the optimizing agent. I now examine his idea in three ways:

1. The market, an impersonalized agent, is acting as an optimizing agent.
2. Some agency, such as military logistics or a private company, is acting as an optimizing agent. This idea leads to management science, not economics.
3. The individual, as a player, is optimizing to achieve his object (end).

Correspondingly, these three cases imply:

1. may be an idea of an ‘invisible hand’, but the definition of the market must be a computer to calculate an optimizing problem. Otherwise, the market plays a game with the industry (factories) as a whole.
2. may be Operational Research (OR), which is actively employed for military operations; in particular, logistics.
3. is a standard theory of microeconomics.

These interpretations form different explanations for the working of the invisible hand. However, this hand is no longer relevant to its original meaning in ordinary life. To put it another way, it is as if the invisible hand has been confined to a small mathematical laboratory to conduct mathematical experiments on ideal situations outside real life.

Depending on the approach adopted, laboratory experiments on optimizing agents may be classified into subsets as either game theoretic or related to general equilibrium. Chapter 2 gives more detail about general equilibrium. Here, I discuss the game theoretic approach, which is applied to levels 1 and 2 in Table 1.3. The game theoretic set contains various historically developed forms, including several cascades derived from von Neumann’s original, such as:

1. Max–min Theorem developing LP, i.e., Linear Programming
2. Balanced Growth Theorem: Nonlinear model and joint productive cycles giving a hint of genetic algorithm GA, Genetic Algorithm
3. Nash Theorem and Core Theorem accompanying Bargaining: Ultimatum game
4. Evolutionary Game Theorem: Replicator dynamics
5. Cournot Theorem and von Stackelberg Theorem: Contract/Bridge and Dynamic Programming

These detailed forms have been described in Aruka (2011, Chap. 1).
1.5 The ‘Invisible Hand’ in the Game Theoretic Approach and the Limit of Computability

The game theoretic view may historically be explained by the classical Max–Min Theorem and its duality theorem of a two-person game or linear programming. This approach was originally established by von Neumann (1937), and extended to a quasi-growth model of balanced growth with multiple sectors operating side by side. In contrast with the linearity of productive processes, the introduction of a growth rate or interest rate makes the model nonlinear, where the objective functions are rates of growth and interest, defined as ratios of the model variables. The model’s constraints are all linear. Von Neumann formulated the balanced growth model of the multi-sectoral production model in terms of the max–min game theoretic form but proved the model equilibrium by means of the Fixed-Point Theorem.

Von Neumann’s game fulfills the max–min property, so the solution does not guarantee a Nash equilibrium. To derive a Nash equilibrium, even restricting the model to a simplified form like Leontief’s input–output model, another interpretation is necessary, which assumes each producer (process of production) is a player (Schwarz 1961). This meaning of the framework is very different from the world of the max–min theorem. Equilibrium can be achieved in either model, but by different mechanisms. It is therefore obvious that the meaning of the invisible hand will also be different in each.

In Robbins’ approach, the market may sometimes be replaced by a planning agency, such as in socialist regimes. In this case, the agency is a kind of computer. As was seen in the debate on planning economies in the 1930s, many economists believed that super-computers would eventually replace the market, which turned out to be completely untrue. We now know computers can replace neither markets nor planning agencies. However, computers are now vital for the successful operation of the market.

The history of computers is quite complex. As Arthur (2009, Chap. 8: Revolutions and Re-domainings) suggested, the financial market emerged rapidly because of the evolution of computers in the last century. The so-called renovation of financial business was then feasible, because computers evolved to solve the complicated risk calculation needed for option dealings (Arthur 2009, p. 154).

In the 1960s, putting a proper price on derivatives contracts was an unsolved problem. Among brokers it was something of a black art, which meant that neither investors nor banks in practice could use these with confidence. But in 1973 the economists Fischer Black and Myron Scholes solved the mathematical problem of pricing options, and this established a standard the industry could rely on. Shortly after that, the Chicago Board of Trade created an Options Exchange, and the market for derivatives took off.

The Black–Scholes equation Black and Scholes (1973) has not been found to have long-term legitimacy in the options market, because of the collapse of long-term capital management (LTCM), to which Myron Scholes was deeply committed. This means that, for obvious reasons, computation of risk cannot necessarily guarantee the correct working of the financial market. Instead of being the market
itself, computers have instead become a means to operate and expand the market. Computation, however, is merely something to which the market must adapt, and is not subject to the market.

1.6 The Invisible Hand in the Self-Organization of the Market

I have previously discussed some instances of the deprivation of ordinary life, and the debate on the benefits of a planning economy in the 1930s. At the time, the views of Friedrich Hayek (1899–1992), who envisaged the limit of the planning economy and computability, were not well regarded, but 40 years later he was awarded a Nobel prize, and many socialist regimes have now broken down. His philosophy has also provided a justification for financial globalization. It is important to appreciate that Hayek (1945) discovered an important function of the market, and revealed the potential exchangeability of supply and demand. This can be visualized as the creativity of the market (Mainzer 2007, VII, p. 230).

According to Friedrich Hayek, markets themselves are centers of creativity, because they serve as procedures to discover the opportunities of goods. The coincidence stands thereby at the beginning. In this sense the market knows more than the individual producers, salesmen and customers who form the market.

Hayek depicted the effects of this function at an earlier stage of the market economy. As history shows, market expansion never creates harmony. In this sense, the invisible hand does not work at all in the later stages of market development. Hayek did not refer to changes in trading technology as a cause of changes to the meaning of the market. I will discuss this further in Chap. 5.

The over-expansion of the market cannot be checked by the market itself in an industrial society, but above all we have seen the over-expansion of the financial market. This can lead to chaos, which is easily discernible in modern financial markets. A series of new financial commodities results from co-mingling option dealings and computation, as Arthur (2009) stated. This accompanies a vertical integration of the market as well as a much higher density of financial circuit.

According to econophysics, the market economy is usually used to produce a power law distribution of price fluctuations, share holdings, capital size, and sales, which is characterized by a ‘heavy’ or ‘fat’ tail. This distribution implies that it holds scale-free property at all points. A kind of polarization evolves rapidly at every step. Consequently, we see strong vertical integration in all market networks. This kind of vertical integration leads to a monopolistic concentration, with just a few winners and many losers. The modern economy can therefore be considered as ‘winner-takes-almost-all’, borrowing from Frank and Cook’s (1995) term ‘winner-takes-all’.

Hayek’s self-organization, triggered by a creative coincidence of the market function, however, tends towards a special distribution with a heavier tail than that of
a Gaussian distribution. Put another way, his self-organized market is often doomed
to develop into a vertically integrated economy, heavily biased towards the financial
elements. Paradoxically, the result does not have an affinity for market efficiency.
A modern financial circuit cannot guarantee either market or system stability. Self-
organization can happen automatically, and a self-organizing process produces more
than the sum of its parts. Drawing on Klaus Mainzer (2007, p. 171), cancer is a kind
of self-organization, whose full evolution implies the eventual death of the human
body. By the same token, self-organization can also lead to the death of the market.

1.7 Invisible Hands and Market Failures

The idea of the invisible hand had just been put forward in 1922, when Edwin
Cannan edited the third edition of *Wealth of Nations* (Smith 1776, 1922, London:
Methuen). In this, Adam Smith explicitly referred to the *invisible hand* in Chap. 2:
Of Restraints upon the Importation from Foreign Countries of such Goods as can
be Produced at Home, in Book IV: Of Systems of Political Economy. Since then,
the term has been over-used. This brief look at the meanings of the phrase ‘invisible
hand’ has therefore largely been in terms of the traditional economic theories. As
an optimizer independent of participants in the market, the ‘invisible hand’ does not
work well in either classical or financial markets.

In Hayek’s view, the market does not operate automatically. Instead, it must
reveal and exploit a set of potential resources, either things or people. As mathe-
matical economists like Arrow and Debreu (1954), Debreu (1959) and Arrow and
Hahn (1971) discussed, we need a bundle of mathematical assumptions to fulfill
the general equilibrium. Such assumptions are too precise to be practical in the real
world. As Bliss (1972) stated, by moving further from reality, general equilibrium
analysis contributed to its own difficulties. The continuity of real numbers is
indispensable for equilibrium. But the real world does not contain $\sqrt{2}$ cars. Instead
of giving up continuity, we can use a discrete algorithm, although this may reach
the limit of computation. We therefore see that there are theoretical obstacles to the
implementation of general equilibrium, and we discuss this more in Chap. 2.

There is a further problem with implementation. The financial market cannot
dispense with supervision from a regulatory body such as the Financial Supervisory
Agency. The contemporary financial system is essentially based on the world
external to its system. The most typical example is that the best risk-free asset
is sovereign bonds. This shows clearly why the financial market could not be an
automatic optimizer. This is the *basic proposition of risk calculation*. This then
is similar to the historical logic of the ‘gold standard’. The financial market cannot
find its own solution but needs a set of regulations to relax the high tension or load

---

7Macfie (1967). He noticed that the popular usage of the term ‘invisible hand’ was driven by
Cannan’s addition of this term to the index.
of trading activities as the amount of trading increases. The market system must pay more for its sustainability, otherwise governments will have to do so. It is therefore easily verified and clear that the individualistic aspects do not hold automatically except for minor markets. I therefore conclude that the most important factor has been the interaction between the international financial market and national economies. Marshall envisaged that a harmonious market could be sustained by the spirit of economic chivalry, but in the modern financial world, we look instead to regulatory institutions.

1.7.1 The Trade-Off Between the Real Economy and the Financial Economy, and the Differential Effects on Each

Finally, I discuss a recent interactive financial process. The story of globalization, since the 1990s, has been around a very limited number of international investment banks. Globalization was driven by over-saving worldwide, typically observed in pension funds, accumulated by the baby-boom generation born just after the Second World War. Over-saving must be compensated for by investments. But investments must be financial. The investment banks therefore invented a series of financial commodities to satisfy this need for investment products.

To do so, they needed a new design for financial capitalism. The selected design was a system of reassurance, which was set in an attractive system to transform loans into assets via investment partnerships outside the official stock exchange. The reassurance market became a symbol of this new form of capitalism, created solely to absorb individual savings. The debt bond swap was actually a classical way to absorb a potential surplus of money, often seen in feudalistic or medieval society. In other words, the idea is obsolete in a sense. Under this new mechanism, a financial network grew rapidly, but this explosive expansion, with its accompanying ups and downs, is closely connected with asymmetrical option trading, which always leads to over-inflated prices. Since modern option trading is on the margins, and without recourse to the main market, we can no longer estimate the true transaction balances. This has come about as a natural result of its institutional settings.

As the collapse of Lehman Brothers showed clearly, private bankruptcies involving large amounts of money need to be remedied by government. This has led to two diverging features: a quick recovery in the financial sector, and slow progress for many national economies. In the US economy and that of other countries, in particular Japan, national savings are falling fast. The United States reached negative national savings in the 1990s, and Japan is rapidly following suit, experiencing negative national savings in the 2010 fiscal year (although a change of accounting operations meant that the amount was formally nearly zero). The financial crisis is having an effect on national wealth on a global scale.

8See Cabinet Office Japan (2010).
There is a trade-off between financial balance and real economic values such as GNP. This is why it is impossible to separate the affairs of any one country from international financial matters. Large amounts of money, exceeding the sum of the US and Japanese GDP, have flowed offshore from domestic economies, although this does not take account of movement of money derived illegally, otherwise known as ‘shadow movement’.  

Habib (2010, p. 15) published some figures of net foreign assets (NFA), i.e., excess returns on net foreign assets.

\[
NFA = \sum BGST + \sum IIB + \sum KG + \sum (EO + KA)
\]  

(1.1)

Here \(\sum BGST\) is the cumulative balance of goods and services including unilateral transfers and compensation of employees; \(\sum IIB\) is the cumulative investment income balance; \(KG\) is cumulative capital gains; and \(\sum (EO + KA)\) is the cumulative errors and omissions plus the capital account (Fig. 1.5).

### 1.8 Some Myths of Modern Economies

There are many myths about modern economies, which can also be considered as stylized facts of capitalism. I will consider three in particular:

1. More liquidity is better.
2. Financial markets are efficient.
3. Economics is an equilibrium system.

I have observed that we have two ways of rejecting myths or stylized facts. One is a criticism of traditional economics, and the other is a criticism in terms of traditional economics, for example referring to the meaning of the financial matching mechanism (see, for example, Helbing’s 2013a,b,c; Helbing and Kirman 2013).

---

9See, for example, Petras (2012): “An economist of impeccable credentials, James Henry, former chief economist at the prestigious consulting firm McKinsey & Company, has researched and documented tax evasion. He found that the super-wealthy and their families have as much as $32 trillion (USD) of hidden assets in offshore tax havens, representing up to $280 billion in lost income tax revenue! This study excluded such non-financial assets as real estate, precious metals, jewels, yachts, race horses, luxury vehicles and so on. Of the $32 trillion in hidden assets, $23 trillion is held by the super-rich of North America and Europe.”
1.8 Some Myths of Modern Economies

1.8.1 Is More Liquidity Better?

If more liquidity continues to be injected to relieve bankruptcies or require more caution in financial investments, the real causes of the loss incurred will never be removed. Increased liquidity interrupts a route connecting a useful fund with another useful purpose.

Lemma 1.1. ‘More liquidity’ means that the financial market does not have its own internal solution.

To escape the crisis, it is necessary not only to clear debts, but also to invent new financial commodities. However, no more loans could be created because of the low expectations of the loan market. These new commodities could therefore only be actualized if suitable funding was injected from outside the market. Investment banks therefore needed to persuade savings banks or building societies holding high amounts of deposits to deliver their funding into trust for the investment banks. In extreme cases, investment banks have urged governments of countries where there are relatively high levels of savings to reform the institutional settings to promote financial investment. As a result, a national economy targeted by the market is artificially depressed by losing options to invest. This is the situation now seen in Japan.
Lemma 1.2. The market always needs external resources. This doctrine has existed unchanged since the industrial revolution.

Funding the relief measures for bankrupted corporations can exhaust even the tax take of a national economy. The relief of AIG after the Lehman shock, for instance, cost over US $150 billion.\textsuperscript{10} In Japan in the 1990s, record sums were spent to bail out banks after excessive loaning during the 1980s bubble period.

This resulted in a chronic and serious record of negative national savings, which must be a danger sign that a national economy may be at risk of collapse. Other European countries may also fall into this situation if the financial crisis expands further.

These treatments correspond to opportunity losses in a traditional economic sense. That is to say, we can criticize this critical situation from a traditional economic standpoint. The chosen remedies may invalidate alternative chances for improvement, but the idea of opportunity costs has never been taken into account.

A supplementary countermeasure was proposed by Silvio Gesell (1862–1930) in Gesell (1916), whose ideas were favored by Keynes and Hayek. His idea was negative interest, called ‘stamp money’. The idea is more usually applied to local or regional money. When stamp money is used, it incurs commission fees (stamps). Stamp money therefore loses one of the main characters of money, and cannot then be used for any financial purpose.

Chiemgauer\textsuperscript{11} studied money in Germany, and found that about a half of regional GDP was achieved purely from regional money, without any resort to national funding Gelleri (2013). This remarkable result shows that national or universal money is not always necessary. This kind of intermediary, rather than financial commodities, might therefore stabilize a local economy to prevent financial invasion or default.

\subsection*{1.8.2 Are Financial Markets Efficient?}

The financial market is largely driven by speculation. So we must consider the efficiency of financial markets. Financial efficiency is truly defined only in terms of the risk calculation. This idea must be distinguished from the classical idea of efficiency. In short, financial traders have nothing to do with their fixed inner preferences, because they have no reason to preserve their preferences for particular kinds of financial commodities. Thus this proposition is rejected by the classical ideas of traditional economics. Looking back on the history of economics, the idea of efficiency is derived from the assumptions that the agent should keep a

\textsuperscript{10}See, for example, the article of Felsenthal and Zuill (2008) appeared in Reuter.

\textsuperscript{11}Chiemgauer is the name of a regional currency, a large-scaled local currency, started in 2003 in Prien am Chiemsee, Bavaria, Germany.
constant list of his/her own preferences, even if he is permitted to estimate others’ preferences and add his own to them. If he should give up working to his own innate preference, his preference would no longer be properly appreciated in the theory of optimization. In other words, first, the domain of preference is given. Second, a maximum set is attained over the objective function (for example, the utility function) constructed on the domain. If the agent leaves the domain, his optimization can no longer be guaranteed.

However, the financial auction presumes that any agent can be or can imitate any other by his transition, typically from seller to buyer or buyer to seller. Sometimes this transition is regarded as the mixed strategy policy. In financial trading, it is meaningless to cling to something that has been especially fixed. So agents’ behavior is free from the subjective criteria of individualistic preferences. The introduction of random preference will not change this.

**Lemma 1.3.** *Classical efficiency must be distinguished from financial efficiency (disclaimer of individualistic fixed preference relationship in the auction).*

I now touch on the technical details of the serious difference between the classical and financial matching mechanisms. In the market, there are two different kinds of order, market order and limit order. Market order is the order created without any bidding or asking price (the contract price is given by the market). Such a distinction may also be allowed in an auction. An important feature of the classical trading system is marked by the rule that an agent never gives up their original preference until the trade session is completed. However, in a financial auction, a trader adopts a mixed strategy of being both seller and buyer, depending on circumstances, so that their original preference is irrelevant. In financial trading, the risk preference on future expected margins will dominate.

**Lemma 1.4.** *The exchange rate for bread has nothing to do with the exchange rate for financial dealings.*

In the classical system, the agent who sets the desired amount of a set of price and quantity to trade with an arbitrary agent must have an independent plan, which is never influenced by the environment but only inner preference. The agent must therefore cancel the original offer if things do not go to plan. Suppose that the agent is a buyer who wants to purchase a desired good whose price is currently lower than he is prepared to pay. In this case, where there is any market order to sell, any purchase plan could be at least partly fulfilled. Here, let the concerned buyer be the person who obtains a contract. As the quoted price is raised to a price higher than the buyer’s planned price, the buyer will give up the contract right to purchase. The buyer must then become a seller of the obtained amount, adding to the market supply. However, this transition from buyer to seller must not be realized in the Walrasian tâtonnement, because all the contracts obtained before equilibrium are canceled.

---

The actual bidding/asking mechanism, i.e., the supply and demand curves, can usually provide the participants with all the information they need. This knowledge is sufficient to activate the agent reactions, which lead to the second stage of coordination. This may no longer be based on the original preference but rather interactive effects.

A buyer could be allowed to change into a seller, or vice versa. In financial markets, buyer or seller is only a temporary state. Notice that the transition from buyer to seller or the reverse must be fulfilled in the financial market. The buyer who temporarily gives up his demand can wait for a future price rise as a potential seller. An agent wishes to make potential extra profit, so he must bet on the right side of a rising price. This is a gamble, but see Aruka and Koyama (2011, p. 149) for the inner mechanism of a double auction.

**Lemma 1.5.** An agent wishes to make potential extra profit, which is a gamble. Its efficiency is not linked to efficiency in traditional economics.

### 1.8.3 Is Economics an Equilibrium System?

Equilibrium in economics is between supply and demand. For simplicity, we will fix the supply side by regarding it as fixed resources. We can then focus on the demand side of the economic equilibrium. In either the classical principles of political economy or neoclassical economics, market efficiency emerges by way of the law of demand. Here we limit demand to ‘consumer demand’. In the ideal thinking of economics, the law of demand can coordinate shortages or surplus in the market, which therefore implies the existence of the ‘invisible hand’.

We denote demand at price \( p \) by \( f(p) \). We then have a formula between two distinct price vectors \( p^1 \) and \( p^2 \):

\[
(p^1 - p^2)(f(p^1) - f(p^2)) \leq 0 \quad \text{i.e.,}
\]

\[
dp df \leq 0
\]

This is the **demand law**, which describes the converse relationship between a demand and its price, as discussed further in Chap. 2.

In classical economics, there is no analytical formulation for this efficiency. However, as neoclassical economics was developed, market demand was analytically designed as a summation of individualistically subjective demands, each of which is derived from a maximization of the subjective utility function. This reasoning, if successful, can provide a justification of market efficiency, in terms of subjective optimization. Each subjective plan to consume by each household is designed to generate a well-defined market demand behavior.

However, this immediately turned out to be incorrect when the Pareto–Slutsky equation was formulated in the first decade of the last century, because the sign of
income effect was not confirmed to derive the consumer demand law. A sufficient condition was not established until Hildenbrand (1994).

**Lemma 1.6.** The consumer demand law requires a sufficient condition that the spread of demand $R(x)$ is greater as income $x$ increases.

Consumer demand microscopically derived does not hold consistently by itself. That is to say, as the income size increases in a given income class, the variance of expenses between consumers in that class should increase. This heterogeneity of consumers can assure the positive nature of the income effect.

**Lemma 1.7.** Advocating microscopic economic behavior cannot generate a consistent macroscopic consumer demand. That is to say, ‘invisible hands’ cannot be guaranteed by individualistic behavior itself (invalidity of indivisible hand).

Thus we cannot rest upon the idea of the invisible hand, which does not work by itself, but needs to be complemented by other measures. One such measure may be an income policy or another policy measure to make up the equilibrium to eliminate the excess or deficit of consumer demand.

### 1.9 Harmony Between *Homo Economicus* and *Homo Socialis*

I have examined the implications of various ideas emanating from popular belief. Needless to say, these beliefs appear to be rooted in ordinary ways of thinking, that is to say, public views on economics. We must therefore escape from a narrow view limited to economics. But we must then understand the meaning of ‘the economy’. In Sect. 1.2, I stated that in medieval times, the term ‘economy’ implied the household of the feudal landlord. I now examine further its historical and linguistic meaning.

The word ‘economy’ can be traced back to a Greek word: *oikonomia*. *oikos* means ‘home’, while *nomos* means ‘law’, hence economy was historically interpreted as ‘wise administration by the patriarch’. The *Oxford English Dictionary* states:

> ORIGIN late 15th cent. (in the sense ‘management of material resources’): from French *économie*, or via Latin from Greek *oikonomia* ‘household management’, based on *oikos* ‘house’ + *nemein* ‘manage’. Current senses date from the 17th cent.

‘Economy’ in the Western sense, back to the ancient Greek, retains the sense of ‘management of material resources’. However, by 2010 in Japan, there was another context in the East, as this excerpt from a press conference statement of a Japanese minister for Financial Services shows:

> As the proverb goes, an economy is all about governing a society and saving its people - every single one of them.\(^{13}\)

---

The proverb mentioned is from the ancient Chinese School of Gongyang. This view emphasizes the first priority as the relief of people in view of *natural justice*. There is insufficient space in this book to discuss the details of ancient Chinese thinking, but it is obvious that the development of a nation’s wealth must depend on fair governance. I will, therefore, *not* discuss increasing wealth by ‘good’ or ‘bad’ means, but rather in a way that is consistent with natural justice. In this sense, we will not pursue a single means of attainment. As *The Art of War* by Sun Zu (515BCE-512BCE) argued, we want the fulfillment of all five of these items:


The word ‘economics’ in Chinese retains an explicit meaning of a *norm of morality*.

经济济民 is abbreviated into 经济 economy (经济学 economics).

In ancient China, there was some metaphysical thought that underestimated personal business activities on the grounds of morality, according to Yao and Shun. For instance, “If a sage were content simply with eating plain food and drinking water, then it would make no sense to perform acts of charity for the people”.

Japanese people, often merchants, tried to accommodate Confucius’ teachings to business activities, enabling them to make a profit without contravening his tenets. For example, Miura Baian (1723–1789) in Miura (2001), a physician of Chinese medicine, wrote a book on economics called *Kagen*. He systematically constructed a positive as well as a normative theory. As his positive contribution, he analyzed price variation in relation to demand and supply, by taking an example of the wage rate fluctuation in terms of excess demand for labor. He was writing at almost the same time as Adam Smith wrote *The Wealth of Nations*, and like Smith, Baian also touched on the norm of morals for economic measures. His moral doctrine was based on the *Analects of Confucius*. He successfully discovered that a country’s wealth is not limited to monetary values, but can also be found in welfare in conformity with justice. In Baian’s view, therefore, policy measures should be concentrated towards the theory of enlightened rule and succor of the people. Profits must not be monopolized, but distributed among the people by the earners themselves, so that accumulation of wealth becomes merely a tool for the essential object of public welfare.

I now turn to Dirk Helbing’s views on the harmony between *Homo economicus* and *Homo socialis*.

---

14This book is estimated to have been written in the Warring States Period (476–221 BCE).
15This citation is from Shibusawa Memorial Museum (ed.) (2009, p. 56).
16See Preface of this book.
1.9 Harmony Between *Homo Economicus* and *Homo Socialis*

### 1.9.1 Dirk Helbing’s View on Harmony in Society

In a recent study, Dirk Helbing, professor at ETH Zurich, concluded that “The social nature of man has dramatic implications, both for economic theory and for the way we need to organize our economy”. As we become more and more connected with others, *Homo economicus*, the independent decision-maker and perfect egoist, is no longer an adequate representation or good approximation of human decision-makers. Helbing’s view is that “reality has changed. We are applying an outdated theory, and that’s what makes economic crises more severe”.

**Outdated Theory, Outdated Institutions.** Helbing believes that social behavior is vulnerable to exploitation by *Homo economicus*. In a selfish environment, *Homo socialis* cannot thrive. In other words, if the settings are not right, *Homo socialis* behaves like *Homo economicus*. Helbing suggested that this may be why it has taken us so long to appreciate the existence of humans as social animals, and further pointed out that economic theories and institutions tend to favor *Homo economicus*, not *Homo socialis*.

In fact, many of today’s institutions, such as homogeneous markets with anonymous exchanges, undermine cooperation in social dilemma situations, i.e., situations in which cooperation would be favorable for everyone, but noncooperative behavior promises additional benefits to individuals. This calls for new institutions.

**New Institutions for a Global Information Society.** In the past, people built public roads, parks and museums, schools, libraries, universities, and free markets on a global scale. What would be suitable institutions for the twenty-first century? Helbing suggested that “reputation systems [could] transfer the success principles of social communities to our globalized society, the ‘global village’”. Most people and companies care about their reputation, so reputation systems could support social decision-making and cooperation, with better outcomes for everyone. In fact, reputation systems have rapidly become widespread on the internet. People rate products, sellers, news, everything, on sites from Amazon to eBay and Trip Advisor. We have become a ‘like it’ generation, listening to our friends’ likes.

Crucially, systems involving recommendations should not narrow down socio-diversity, as this is the basis of happiness, innovation and societal resilience. Helbing stressed the importance of remaining in touch with an objective picture of the world, and not living in a ‘filter bubble’, Eli Pariser’s term for a world where important things get filtered out in favor of what’s fun. Reputations systems should therefore be pluralistic, open and user-centric. Helbing (2013c, p. 3) noted that “pluralistic reputation systems are oriented at the values and quality criteria of individuals, rather than recommending what a company’s reputation filter thinks is best. Self-determination of the user is central”.

---

17 See Pariser (2012).
Fig. 1.6 Viscount Eiichi Shibusawa. Cited from Wikipedia: http://en.wikipedia.org/wiki/Shibusawa_Eiichi

1.9.1.1 Shibusawa’s View of Harmony in Society

Helbing’s idea is compatible with the ideas of Viscount Eiichi Shibusawa (1840–1931) (Fig. 1.6), a father of Japanese capitalism writing during Japan’s industrial revolution, the Meiji Restoration. Throughout his life, Shibusawa pursued harmony between ethics and the economy, recognizing Adam Smith’s moral philosophy, but basing his thinking on a different source of philosophy. Shibusawa always tried to apply Confucius’ *Analects* to his thinking on the harmony between ethics and the economy. He was able to fund or support 500 private business enterprises, most of which are now major companies. He also involved himself in 600 projects to set up educational institutes and provide social welfare, including Hitotsubashi University.

Shibusawa’s belief was not in line with traditional Confucian philosophy in the sense that eating plain food, drinking water, resting, my bent elbow as a pillow—therein, too, lie my pleasures. Shibusawa cited this in a speech and went on to say:

> Taking these words very literally, one thinks that Confucius does not care about fame and wealth. But such an interpretation ignores the deeper meaning of the phrase “therein, too, lie my pleasures”. Precisely because the sage treasures justice or righteousness, he finds great pleasure in the simple life as well. Confucius does not regard only plain living as his pleasure. ··· [C]ultivated people need not be ashamed of the pursuit of profit in accordance with righteousness. According to what I have read and heard, ··· Adam Smith laid the foundations for modern economics by setting forth an ethic of sympathy. ··· I believe that Confucius and Adam Smith share the same idea, which is the [co-existence of] profit and righteousness. This is a universal principle that holds in both East and West.

Shibusawa’s views lead to the next lemma:

The leadership of a country cannot afford to overlook the importance of enchanting industry and making profit.

---

Summing up, he called his synthesis of business ethics and profit-making under fair governance in view of public benevolence **harmony between ethics and the economy**. This harmony is the same as Helbing’s harmony between *Homo economicus* and *Homo socialis*.

In this book, I therefore suggest replacing Shibusawa’s description of coordination between ethics and the economy with that between *Homo socialis* and *Homo economicus*, in line with Dirk Helbing. Shibusawa’s insight means that from the very beginning of Japanese capitalism, an effort was made to integrate *Homo socialis* and *Homo economicus*, which is also why I have not sought to talk only about *Homo economicus*.

# 1.10 The Mozi School: Impartial and Heterogeneous Interacting Agents

The more usual understanding of Confucius’ teachings is not only rather different from Shibusawa’s advanced interpretation, but also lacks social views of institutional and behavioral interactions and coordination. This is often considered a fatal flaw of Confucianism, and requires another doctrine to complement it.

At almost the same time as Confucianism developed, so did the Mozi school, founded by Mozi (ca. 470 BCE-390 BCE) and espousing **Mohism**. The two schools shared some ethical views: Mozi is recognized as a pacifist, but one who accepted the need to take action against invaders’ attacks. Mozi’s high reputation internationally is because of his constant passion for the good of the people, without concern for personal gain or even his own life or death. His tireless contribution to society was praised by many, including Confucius’ disciple Mencius. He acted on the belief of **impartial caring** all his life. Mozi, Book 7, Will of Heaven 1 in *Mohism* in Mozi (1934) wrote:

> He who rules a large state does not attack small states: he who rules a large house does not molest small houses. The strong does not plunder the weak. The honored does not demean the humble. The clever does not deceive the stupid. This is beneficial to Heaven above, beneficial to the spirits in the middle sphere, and beneficial to the people below. Being beneficial to these three, it is beneficial to all. So the most excellent name is attributed to such a man and he is called sage-king.

Confucianism gives its highest level of priority to propriety then to the arts. However, the Mozi school was organized by engineers and artisans, whose respect was naturally given to **actual practice**. The Mozi school was therefore particularly excellent in its social and political commitments. Much more interestingly, it developed a unique moral philosophy. Mozi believed that a large power should not attack a small country, and coupled his belief to practice. He always tried to exercise

---

19 Mozi was a carpenter and was extremely skilled in creating devices, and recognized as an expert on fortification.
his professional expertise in munitions and fortifications to protect smaller countries from attack by larger ones. For example, he successfully prevented the state of Chu from attacking the state of Song by engaging in nine simulated war games, defeating Gongshu Ban, a strategist employed at the court of Chu.

I now turn to the utilitarianism of heterogeneous interacting agents. The idea of impartial caring is often considered to be the same idea as Western utilitarianism. However, Mozi utilitarianism was not based on individualistic homogeneous agents. Instead, Mozi had a clear concept of multiple stakeholders or heterogeneous agents. The school then evolved Mozi’s original idea away from the earlier version, because it was too restrictive to accept the idea of pursuit of profit, and advocated a system of mutual benefit by taking into account that profit can be generated in a way that allows others to generate a profit too. Hence the system of mutual benefit aims to create a situation where everyone can enjoy the welfare of the whole world, which would arise at the point at which pursuing self-interest at the cost of others stopped, according to the spirit of impartial caring (Asano 1998, p. 59).

The school did not focus on the so-called naked individuals when it developed the idea of impartial caring. It regarded the world as the aggregate of seven different intersecting units (father-sons, elder brother-younger, lord-subjects, myself-others, wife-other wives, family-other families, home country-other countries): within each unit, unit members share the same interest, and between units, they stand in interest opposition. … The Mozi school classified the world in the light of only one criterion to maintain social order, according to its original purpose of impartial caring. … It asserted that there was no discrimination between his own unit and an opponent unit (impartial); an agent should not attain his own benefit at the price of his opponent’s (caring or others regarding). 21

It is clear that the Mozi school encompassed all the constituents for modeling the heterogeneous interacting agents:

• The community members are functionally classified into heterogeneous agents.
• Heterogeneous agents are aggregated into subgroups as clusters so that an agent belongs to a cluster in which unit members’ interest is entirely unanimous, but the clusters are mutually conflicting.
• Profit may be exchangeable between clusters within a system of mutual benefit. This suggests that agents are also exchangeable.

As we have seen, Mozi propounded the idea of multiple stakeholders, now the situation for the modern corporation system, in terms of the old idea of heterogeneous agents, and also added an original idea of moral belief as a coordination mechanism. In this kind of framework, human interactions are understood in terms of a macroscopic microeconomic feedback loop and take into account a kind of macroscopic condition. Thus his disciplines proposed the following proposition:

---

20 The state of Chu existed from around 1,030 BCE to 223 BCE.
Proposition 1.1 (Aruka 2007 in Aruka (2011, Part II)). The framework of heterogeneous interacting agents can employ the system of impartial caring and mutual benefit as its moral code.

Incidentally, Shibusawa’s social view and ethics cannot hold without a conjugation with the dynamic coordination among heterogeneous clusters innate to Mohism. In some senses, therefore, Shibusawa succeeded in integrating Confucianism and Mohism.

1.11 Human Interactions: A Macroscopic Microeconomic Feedback Loop

The master equation used in quantum physics, i.e., a state transition equation in physics, implies the existence of an in-flow and out-flow rate, describing the forces of attracting and repelling. These rates can be implemented by the multinomial logit typed utilities, which is discussed further in Chap. 2. The equation can be used to depict the successive processes of social decision-making reflecting different kinds of heterogeneous interaction. This is basically the economic implication of sociodynamics in terms of moral science, developed by Weidlich–Haag in Stuttgart in Weidlich and Haag (1983), and means that we can argue the formation of solidarity in connection with a genuine historical path. In other words, sociodynamics, in terms of the system of mutual benefit in line with Mohist thought, implies the feasibility of numerical analysis, instead of the system of representative agents equipped with invisible hands.

As society becomes more complicated, the more the market environment itself evolves voluntarily. It seems strange to believe a modern form with the same name must guarantee the same function on an evolved system, but more efficiently. A smart grid system may be one such modern form. Following Mainzer (2010, pp. 219–219), I illustrate a typical smart grid system.

Smart grids combine energy systems and information and communications technology as a symbiosis. One of the problems with wind wheels and solar cells is the unpredictability of production. In intelligent networks the need in high degree can be satisfied locally. In addition, assume the following model with concrete basic ideas:

1. The demand exists either within a local (regional) subnet or between subnets. Only in exceptional cases is a reserve capacity taken up. See Fig. 1.7.
2. Energy reconciliation takes place between different levels instead of or between balanced groups on the same level.
3. Producers are also consumers, and vice versa.
4. Negotiations over local current supply are accomplished automatically by producer and consumption agents. They are coordinated by the accounting grid manager (balanced group manager), who works in a time-synchronized way on each level. Figure 1.8 shows three such balanced levels.
5. In the model, the negotiations begin every 0.5 s, and have a clear end-point. Bids arriving in the meantime are negotiated in the next period.
**Fig. 1.7** A smart grid circuit. Cited from Mainzer (2010, Fig. 20) with slight modifications.

**Fig. 1.8** A smart grid network model. *a* BGM is the Balancing Group Manager. *b* The model shows three regional BGMs as well as a **layered structure** originating from BGM 110 kv. *c* Except for the traditional circle {producerT, consumerT1, consumerT2}, the remaining part corresponds to the smart grid system.
6. At the beginning of each period, each customer determines whether they wish to participate as a producer or consumer or not join in the negotiations, according to the current situation.

7. Sales offers and bids happen within price frameworks, considering redemption and maintenance costs. There are no long-term contracts with discounts for large and future purchases, which often arise in reality.

As seen in Fig. 1.8, this model forms a layered network, in which we cannot assume a plane-like homogeneous field. First, the negotiation within the smart grid system is by genetic algorithms. In later chapters, I will discuss genetic algorithms further, as well as artificial intelligence (AI). \( 0.5s \) is outside human control. In standard economics, all that matters in negotiations is whether the time is logical. In the real world, however, the practical time taken for processing must matter. A delay in processing indicates inferiority. The smart grid system is managed by the negotiation algorithm, a kind of genetic algorithm, not by humans, as humans cannot operate at machine speed. But negotiation in a game theory situation is different from the negotiation algorithm by AI. Even if standard economics could include optimization in the negotiation process, its algorithm would require a loser, because the idea of optimization is not valid in the landscape of complex interactions of heterogeneous agents at high speed and/or frequency. The same problem also applies to the stock exchange. The actual world dispenses with optimization. Because it keeps moving rapidly, there is no need to fix the present to a particular equilibrium. The stylized facts of economics do not correspond to reality, and in particular to the emerging ICT society. The economic agent must be independent to pursue his own private profit. It is impossible to imagine such an independent node if we regard society as an evolving network. A pressing subject must be the design or construction of the network.

Much of the academic community retains some stubborn belief in invisible hands. The efficient market hypothesis (EMH) is a typical variant of this belief. For example, faced with the global economic crisis, Robert Lucas, a Nobel laureate, in defending economics (The Economist print edition, August 6, 2009), asserted that the current crisis strengthened the credit of the efficient market hypothesis. As Chen (2008, p. 84) pointed out, however, the fundamental assumption behind the EMH is that financial markets are ruled by random walks or Brownian motion. If this theory were true, then the markets would be very unlikely to have large price movements like financial crises. Orthodox economics may no longer derive any prescription for this crisis from its own traditional theory (Chen 2013, pp. 9–10).

It therefore seems important that we should disengage from the EMH swiftly. Current financial markets are filled with substantial secondary noise that could offset primary efforts to regulate demand and supply. Real financial markets are surrounded by too many non-market factors typical of the traditional stock exchange. The so-called sub-prime crisis was essentially irrelevant to the traditional stock exchange system, since it happened outside it, although influencing it heavily and therefore amplifying fluctuations. For example, in the currency exchange market, it is quite normal for professional traders to have mutual exchanges using private information, even though this seems like insider trading.
1.11.1 Economics of a Master Equation and Fluctuations

The real-world economy actually seems to be irrelevant to the EMH. The interaction of heterogeneous factors inside and outside markets may generate many complicated outcomes for the world economy. This resembles the movements of exchangeable agents in a combinatorial stochastic process like the urn process. The stochastic evolution of the state vector can be described in terms of the master equation equivalent to the Chapman–Kolmogorov differential equation system. The master equation leads to aggregate dynamics, from which the Fokker–Planck equation can be derived, so we can explicitly argue the fluctuations in a dynamic
system. These settings are made feasible by classifying agents in the system by type and tracking the variations in cluster size. In Figs. 1.9 and 1.10, I show the images of a master equation as well as stochastic trajectories, probability distribution and quasi-mean values in a phase transition from mono- to bi-stability.

References


Bliss CJ (1972) Prices, markets and planning. Econ J 82(325):87–100


Shibusawa E (2009) Shibusawa’s speech at the Imperial Institute of Invention and Innovation, June 13, 1923 in Shibusawa Memorial Museum (ed.) Guide to the Exhibits Shibusawa Memorial Museum, Tokyo


Smith A (1776) An inquiry into the nature and causes of the wealth of nations. Full text from library economics liberty: http://www.econlib.org/library/Smith/smWN.html


Smith A (1776) An inquiry into the nature and causes of the wealth of nations. Full text from library economics liberty: http://www.econlib.org/library/Smith/smWN.html


Chapter 2
The Historic Design of the Demand Law and Its Reconstruction

Abstract  The historical development of economic theories suggests that the most essential constituents of economics are the demand law, the utility function, the production function, and general equilibrium. These issues were argued professionally from the 1930s to the 1950s, mainly by mathematicians and physicists. The most fundamental of these seems to be the demand law. Many economists have been unable to find a consistently self-contained model either by any kind of individual utility formulation or the revealed preference axiom. This problem was solved by Hildenbrand (Market demand, Princeton University Press, Princeton, 1994), taking into account macroscopic order. Even the consumer theory was too restrictive to encompass many important aspects of consumption activities. In this sense, the traditionally narrow interest may be dangerous because other decisive factors contributing to consumption activities may be missed. There are many aspects to consider, including the inter-connected factors between different income classes and household demands. Household demand includes some items that are so indispensable that demand for them is unaffected by price. Ignoring price levels, people would choose items to meet their desire for both luxury and sophistication. This chapter argues a particular scenario where different forces may apply to consumption activities in different income classes. By focusing on a self-organizing pattern of consumption, we analyze a new facet of interactive correlations among heterogeneous consumers. This may lead us to observe another hidden force driving consumption. Before discussing the detail, I consider the basic structure of traditional theories of static and random preference.

2.1 Some Criticisms of a Utility Function for the Design of Household Demand

2.1.1 Consumption As a Compromise Between Self-regarding and Other-Regarding Interests

People need to consume to meet both subsistence and cultural desires. However, such consumption is always vulnerable to unexpected events. We have grounds to
believe that household demand is not simply the sum of individual demand because families share desires and consumption. We assume instead that consumption behavior of each unit of a household is a compromise between self-regarding and other-regarding interests. Many economists have argued that other-regarding interests are indispensable for ensuring that consumption conforms to social norms. As Aruka (2004a; 2011, Chap. 7) pointed out, humans tend to adopt an attitude of other-regarding behavior. Even if the household is composed of a single person, we should still not jump to the traditional idea of pure individual subjectivism. In general, the person cannot be considered as a self-regarding machine restricted to his innate original preferences. He has his own social status, so will be induced to take into account his learning experiences to adapt to socially arranged modes of consumption. His consumption pattern cannot purely operate on his innate preference list, or be separated from the external world.

However, historically, the design of the demand function has very much focused on individual subjectivism. In other words, the market demand is naively assumed to consist of the sum of all individual demands directly derived from individual utility maximization. The first approach to developing an individual utility function, designed to derive an individual demand function, is almost religious idealism. James Mill, who contributed to the development of neoclassical economics, regarded the innate original preference as “permanent personal identity”, or the mind in itself. This leaves no room for any state-dependent decisions. In terms of standard economics, therefore, the mind in itself may be replaced with intrinsic rationality. Its feasibility could be guaranteed by “free will”, though emotional and ethological reasons can temporarily invalidate individual rationality. Intrinsic rationality, in the classical sense of the mind in itself, was historically argued to have irrefutable properties, which could not be checked in experiments because rationality belongs to some divine world, logically irrelevant to the real one.¹

In standard economics, intrinsic rationality has the following properties for consumer preference:

1. reflexivity
2. completeness
3. transitivity
4. continuity
5. monotonicity
6. convexity

The first four properties are necessary to guarantee the utility function, and some weak convexity in the preference relation is acceptable. The so-called indifference map described in textbooks on microeconomics is not indispensable. Mathematical economics in the twentieth century discovered that there is isomorphism between the preference relation and the utility function. This was innovative, as long as intrinsic rationality is universal. The utility function is defined as equivalent to

¹Modern experimental economists instead treat intrinsic properties as practically operated.
preference ordering in the domain of multiple commodities. The utility function in economics can usually give a unique value to each set of commodity baskets through an affine transformation. These properties guarantee possible preference relations to the maximal elements to guarantee the individual utility function.

### 2.1.2 The Discrete Choice Model of Different Modes

Before we proceed to discuss the demand law itself, I turn to an alternative choice model or utility formulation, leaving the individual stage. Concentrating on intrinsic preference ignores the construction of chosen preferences in a group. Without knowing about the whole of society, we cannot predict the market demand as a whole. It would be very difficult to attain a macroscopic view of demand simply by aggregating individual rationality. An alternative model was therefore proposed by Luce (1959), and is often called the multinomial logit model. It has been widely applied to practical estimation in many industries. Daniel Little McFadden, who was working on this model, was awarded a Nobel Prize in 2000, marking the fact that this is now considered an important economic model. See McFadden (1984).²

In Luce’s original model (Luce 1959), there is a finite number of alternatives that can be explicitly listed. Suppose that there is the universal choice set \( U \). The subset of this that is considered by a particular individual is called the reduced choice set \( C \). A subset of \( C \) is denoted by \( S \). It then holds: \( S \subseteq C \subseteq U \). If we take an example of the problem of choosing dinner, where the decision-maker is faced with either \{Japanese \( \alpha \), Chinese \( \beta \), or French \( \gamma \} \) dishes, the choice set will be:

\[
C = \{\text{Japanese}\alpha, \text{Chinese}\beta, \text{French}\gamma\}
\]

The probability of choosing \( S \) from the set \( C \) is \( P_C(S) \). A subset \( S \) that contains a particular alternative \( \alpha \) may be

\[
\{\text{Japanese}\alpha, \text{Chinese}\beta\} \text{ or } \{\text{Japanese}\alpha, \text{French}\gamma\}
\]

This is therefore described as the binary logit probability.

1. When there is a dominant preference, in other words that \( \beta \in C \) exists, such that \( \beta \) is always preferred to \( \alpha \), it holds that the probability of choosing \( \alpha \) is set as 0; that is to say, \( P_{\{\alpha, \beta\}}(\alpha) = 0 \). Here the removal of \( \alpha \) never affects the choice of \( \beta \) in the remaining set. This may be extended:

\[
P_C(S) = P_{C \setminus \{\alpha\}}(S \setminus \{\alpha\}) \tag{2.1}
\]

²Ben-Akiva and Lerman (1985), Anderson et al. (1992) are also good literatures for discrete choice model.
2. When there is no dominant preference, no absolute choice is ascertained. There may be an alternative $\alpha$. It then holds that $0 < P_{\{\alpha, \beta\}}(\alpha) < 1$. It is usually assumed that the choice probability is independent of the sequence of decisions; that is:

$$P_C(\alpha) = P_C(S)P_S(\alpha) \quad (2.2)$$

As Luce (1959) proved, this condition is necessary and sufficient to derive the relation:

$$P_S(\alpha) = \frac{V(\alpha)}{\sum_i V(i)} \text{ for all } S \subseteq C \quad (2.3)$$

This confirms the existence of a real function with a value $V : C \rightarrow \mathbb{R}$. It is also verified that function $V$ is unique up to a proportional factor $\lambda \in \mathbb{R}$. That is to say, it holds that:

$$V(i) = \lambda V'(i) \text{ for any alternative } i \in C. \quad (2.4)$$

The function $V$ is a Luce-type utility function. Now we substitute $e^{\mu V_{\alpha}}$ for $v(\alpha)$, and $e^{\mu V_{\beta}}$ for $v(\beta)$. It immediately follows that:

$$P_{\{\alpha, \beta\}}(\alpha) = \frac{e^{\mu V_{\alpha}}}{e^{\mu V_{\alpha}} + e^{\mu V_{\beta}}} \quad (2.5)$$

This gives the probability of the decision of $\alpha$ in the subset $\{\alpha, \beta\}$. This expression is useful for estimating an actual mode choice in a group.

We regard a mode as a set of similar types. The mode of a Chinese meal contains many types of dishes with different qualities. A mode may therefore be regarded as an aggregate commodity, by eliminating differences of type due to empirical reasons. Instead of this kind of rounding difference, we could take into account the mode factors (variables) in terms of common measures like time or cost (price).

This idea is immediately applied to an interpersonal framework for choice distribution in society. In our new environment, we are used to several alternative modes for a particular choice, as well as some alternatives of choice qualities. In short, an individual is allowed to choose a particular mode with his preferred quality.

---

3 The difficulty of identifying types may be considered metonymy. Because of the limitations of empirical data, we must abandon a microscopic approach to types. Instead, we employ some mode/state property. Another example is income distribution. “Middle class” is a state property, if we properly define the income range of the middle class. We may then neglect the ingredients of various types of middle-class income. We call such a simplification metonymy from types to state properties, according to Hildenbrand (1994). We employ exchangeable agents instead of precise types. Types are replaced with state variables. These then become surrogates of type, and we focus on a combination of the total number of states and state variables.
In our cuisine mode example, we can consider a simple numerical example of two alternative modes and three people. The meal could be either of two alternative modes \{mode 1, mode 2\} = \{Japanese, French\}. We then employ the univariate case, i.e., cost as a single variable. In other words, we presume the deterministic portion of our utility function as follows:

\[ V_{\text{mode}_i} = \beta^* \times \text{cost of mode } i \]  

(2.6)

The three people are then faced with a list of different prices (cost variable) in each mode (see Table 2.1).

According to Luce’s model, a probability \( P_{ij} \) is interpreted as the choice probability for a particular person of choosing a menu type \( j \) at mode \( i \). Hence it follows that:

\begin{align*}
\text{Individual 1 } P_{11} &= \frac{\exp(50\beta)}{\exp(50\beta) + \exp(40\beta)} = \frac{1}{1 + \exp(-10\beta)} \\
\text{Individual 2 } P_{22} &= \frac{\exp(30\beta)}{\exp(20\beta) + \exp(30\beta)} = \frac{1}{\exp(-10\beta) + 1} \\
\text{Individual 3 } P_{23} &= \frac{\exp(30\beta)}{\exp(40\beta) + \exp(30\beta)} = \frac{1}{\exp(10\beta) + 1}
\end{align*}

(2.7) \quad (2.8) \quad (2.9)

Given Table 2.1, the log-likelihood expression for this sample will be:

\[ \text{LogLikelihood} = \sum_{i=1}^{2} \sum_{j=1}^{3} \delta_{ij} P_{ij} \]

\[ = \log P_{11} + \log P_{22} + \log P_{23} \]  

(2.10)

We then look for the maximum of this function of \( \beta \), found at \( \beta^* = 0.0693147 \). This is judged as the most plausible in the observed distribution of mode selections in society. Hence we substitute \( \beta^* \) into the expressions 2.7–2.9 to get the probabilities (utilities) of mode choice for particular individuals.

---

4 A multivariate case, for example, might use both time and cost as variables.
5 \( \delta_{ij} \) is Kronecker’s \( \delta \).
These arguments have been of much practical interest to policy makers. For example, the choice of travel modes problem is very well known. Given the possible means of transport, \{train, bus, plane, car\}, a traveler can solve the problem of the desirability of each possible mode to travel to his destination.

To make utility maximization feasible, these conditions are required on the deterministic part $V_\alpha$:

**Reproducibility** (consistency): This implies the same choice selection will be made repeatedly under identical circumstances.

**Acyclicity** (a weaker transitivity): This implies a weaker transitivity on preference.

\[ c_1 \geq \cdots \geq c_n \text{ and } c_n \succ c_1 \]  
\[ (2.11) \]

Each error term is independent. We also require independence from irrelevant alternatives (IIA).

\[ \frac{P_i}{P_k} = \frac{\exp V_i}{\exp V_k} = \exp (V_i - V_k) \neq 1 \text{ for any pair } (i, k) \]  
\[ (2.12) \]

which indicates its independence from the number or attributes of other alternatives in the choice set. In other words, each line must not be mutually proportional among different modes.

The limitation of the multinomial logit (MNL) model then results from the assumption of the independence of error terms in the utility of the alternatives. Assume that the choice probabilities (for an individual or a homogeneous group of individuals) are 65%, 15%, 10% and 10% for drive alone, shared ride, bus and light rail, respectively. If the light rail service were to be improved in such a way as to increase its choice probability to 19%, the MNL model would predict that the shares of the other alternatives would decrease proportionately, decreasing the probability for the drive alone, shared ride and bus alternatives by a factor of 0.90 (Koppelman and Bhat 2006, p. 165).

### 2.1.3 Some Generalizations on Random Terms, Heterogeneities, and Social Interaction

We can decompose the utility on the alternative $\alpha$ into the deterministic part $V_\alpha$ and the random term $\epsilon_\alpha$, which then allows us to develop the utility function $V$ to a more general $U$.

\[ U_\alpha = V_\alpha + \epsilon_\alpha. \]  
\[ (2.13) \]

---

6This part was firstly written in Japanese: Aruka (2004b, pp. 163–166).

7See, for example, Durlauf (1997).
2.1 Some Criticisms of a Utility Function for the Design of Household Demand

In the choice set is \( C = \{ \alpha, \beta \} \), the binary choice. This might, for example, mean an individual \( i \) choosing to be either a musician to guarantee the utility \( V_\alpha \) or a doctor to guarantee \( V_\beta \). However, this choice is likely to be affected by events; for example, attending a gig by a favorite band. Each alternative will be influenced by the random terms \( \epsilon_\alpha, \epsilon_\beta \). The binary choice probability of \( \alpha \) is then given by:

\[
P_{\alpha, \beta}(\alpha) = P[U_\alpha \geq U_\beta] = P[V_\alpha - V_\beta \geq \epsilon_\alpha - \epsilon_\beta]. \tag{2.14}
\]

If we limit the binary choice, we can assume that the next choice set will be:

\[
\omega \in \{-1, 1\} \tag{2.15}
\]

A particular choice will be realized by a probabilistic distribution of random shocks. We assume a logistic distribution within a particular \( z \) of the probability of the random shocks on a binary choice \( \epsilon_i(-1) - \epsilon_i(1) \) for individual \( i \).

\[
\Pr(\epsilon_i(-1) - \epsilon_i(1) \leq z) = \frac{1}{1 + \exp^{-\eta_i z}} \tag{2.16}
\]

On the choice plane \( X_i \) for \( i \), we distinguish the observable characteristics from the unobservable heterogeneity \( \eta_i \). Here \( \eta_i = \eta(X_i) \geq 0 \). The density function of the logistic distribution is symmetrically bell-shaped. As \( \eta \) becomes larger, the probability that the difference in utility falls within a certain value of \( z \) decreases (Fig. 2.1).

Individual \( i \) has his own belief about the choices of the other members of the group \( I \) based on information received, \( F_i \):

\[
F_i : \mu_i^\omega(\omega_i \mid F_i). \tag{2.17}
\]
Individuals are inclined to either conform to or deviate from the choices of others in the group.\(^8\) When all the members make the same choice, social utility for any one individual is zero. When they all deviate, according to the Ising model, the social utility for agent \(i\) is negative:

\[
S(i, \omega, \mu_i) = -E_i \left( \sum_{j \neq i} J_{ij} \left( \frac{\omega_i - \omega_j}{2} \right)^2 \right).
\]  

(2.18)

The weight of the interaction \(J_{ij}(X_i)\) on the domain of observable characteristics \(X_i\) is denoted by \(J_{ij}\), relating the choice for \(i\) to that for \(j\). \((\omega_i - \omega_j)^2\) will then give the conformity effects. Social utility \(S\) is a subjectively expected value based on individual \(i\)’s belief about the distribution of social interaction \([J_{ij}(X_i)]\).

The decision process of individual \(i\) is therefore:

\[
\omega_i = \arg \max \{V(\omega, X_i, \mu_i, i)\} \text{ for } \omega \in \{-1, 1\}
\]

(2.19)

It is easy to derive the solution \(\omega_i\) by linearization of utility function \(U\). We assume that:

\[
U(\omega_i, X_i) = h_i \omega_i + \kappa_i
\]

(2.20)

\(h_i\) and \(\kappa_i\) are such that:

\[
h_i + \kappa_i = u(1, X_i), -h_i + \kappa_i = u(-1, X_i).
\]

(2.21)

Since the random shocks are assumed to be subject to the logistic distribution, the probability distribution of the choice of individual \(i\), i.e., \(\omega_i\), can be solved:

\[
\mu(\omega_i \mid X_i, \mu_i) = \exp(\eta_i h_i \omega_i + \sum_{i \neq j} \eta_i J_{ij} \omega_i E_i(\omega_j))
\]

(2.22)

The joint probability distribution of the whole population of the choice of \(\omega\) is then:

\[
\mu(\omega \mid X_1, \ldots, X_n, \mu_1, \ldots, \mu_n) = \prod_i \mu(\omega_i \mid X_i, \mu_i^n)
\]

\[
\propto \prod_i \exp \left( \eta_i h_i \omega_i + \sum_{i \neq j} \eta_i J_{ij} \omega_i E_i(\omega_j) \right)
\]

(2.23)

This solution is underpinned by certain limited assumptions including linearization of the utility function and a special form of the error distribution. By introducing

---

\(^8\)The lower suffix \(-i\) indicates those members other than \(i\).
randomness, heterogeneity, and social interaction, however, it may be possible to predict a macroscopic motion to abandon individual subjectivism, moving away from a shortsighted focus on intrinsic preferences. In Fig. 2.2, we show the framing image of random preference. Intermediating by a “lottery system”, we may be taken into either parallel world.

2.1.4 Some Essential Differences Between Frames

If we focused on the introduction of uncertainty, we would touch on some essential differences. We may formulate several possible frames of preferences under different conditions of uncertainty:

A. State-dependent preference

• A1: If it is hot, the individual prefers to drink milk.
• A2: If it is not hot, the individual prefers tea.

B. Random preferences

• B1: If it is hot, the individual is more likely to prefer milk.
• B2: If it is not hot, the individual is less likely to prefer tea. If it is cool, the individual is more likely to prefer tea.

---

9I have discussed the details elsewhere. See, for example, Aruka (2011, Chap. 8).
C. Socio-dynamic view

- **C1**: If it is hot, many prefer milk but some prefer tea.
- **C2**: If it is neither hot nor cool, many prefer tea and a few prefer milk.

Random preference, as in $B1\&2$, is directly applied, because all the utilities in the model are deterministic and only externally subject to random shocks. Heterogeneous interaction in the context of the Ising model from physics is only introduced in terms of social utility. Heterogeneous interactions are extrapolated to individual utility. Private utility is not affected by social utility, and will never be reflected by heterogeneous interaction rather than by noise interference. Random preference, at first glance, indicates $B1\&2$. The idea of a subjective agent is essential to both $A$ and $B$, and is one who aims to optimize their subjective object. We may, however, create another interpretation by framing the case differently, as such an agent is not necessary in case $C$, moving us away from individual subjectivism. Figure 2.3 illustrates an essential difference between the two frames.
2.2 Analytical Examination of the Demand Law

I turn now to the demand law. The traditional derivation of market demand hinges on the idea of constancy of the “mind in itself”. This belief is the crucial assumption in justifying the universality of the individual utility function. Individual optimization can hold if, and only if, this assumption is retained. However, the constancy of the “mind in itself” is merely a metaphysical presumption taken as irrefutable where “free will” is ubiquitous. In reality, there is considerable evidence to refute the demand law. For example, income effects cause so-called Giffen effects. As Mizuno et al. (2010) and Mizuno and Watanabe (2010) showed, however, even in online markets, which can be considered to fulfill “perfect competitiveness”, customers do not follow the demand law. They do not necessarily increase demand when the price is lowered. Giffen’s discovery, made at the end of the nineteenth century, is still valid today.

2.2.1 A Makeshift Idea of Compensated Demand and Income

We denote a demand at price $p$ by $f(p)$. There are then two distinct price vectors $p^1$ and $p^2$:

$$(p^1 - p^2)(f(p^1) - f(p^2)) \leq 0$$

(2.24)

i.e.,

$$dpdf \leq 0$$

(2.25)

This is the demand law. When an income variation $x$ derived from price changes is taken into account, the tentative formulation is called the Pareto–Slutsky equation:

$$\frac{\partial f_j}{\partial p_k} = \frac{\partial h_j}{\partial p_k} - \frac{\partial f_j}{\partial x} f_k$$

(2.26)

In other words:

- demand change = substitution effects + income effects

$j$ and $k$ are indices of goods. $f_j$ is the demand for good $k$. $h_j$ is the compensated demand for good $j$. $x$ is income level. Demand for a good depends on the price of goods $p_k$ including the good itself ($p_j$) and also on income. Income $x$ is measured in terms of goods. Therefore, income may be changeable depending on price variations $\Delta p$. A change in income naturally induces a change in demand. However, the sign of a change in income is not decisive. This is a complicating factor in establishing the demand law. Consequently, economists were not successful in confirming the
The Historic Design of the Demand Law and Its Reconstruction

sign of the income effect until a new assumption was made by Hildenbrand (1994). It therefore took about 90 years to solve this problem.

Compensated demand is a sophisticated idea. This demand $h$ always has a negative sign with respect to a price rise. $h$ is a special form that only reacts to price variation but not to income level, to guarantee the same level of satisfaction by supplementing a new injection of income if short or reducing it if long.

We introduce into the Pareto–Slutsky equation a very small perturbation of $\Delta p$ and $\Delta h$. A variation of $dp \, df$ caused by $\Delta p$ and $\Delta h$, i.e., $\Delta p \Delta f$ may be approximately estimated as $\Delta p \frac{\partial f}{\partial p_k} \Delta p$. In other words, it holds that:

$$\Delta p \frac{\partial f_j}{\partial p_k} \Delta p = \Delta p \frac{\partial h_j}{\partial p_k} \Delta p - \Delta p \frac{\partial f_j}{\partial x_k} \Delta p f$$

(2.27)

There is no reason for the first item to always be equal to the second. If the second item is negative, the total variation caused by $\Delta p_k$ could be positive, leading to the reverse of the demand law. The Pareto–Slutsky equation, as it stands, is therefore not able to guarantee a definite analytical relation. A so-called Giffen effect cannot be removed.

In Fig. 2.4, we can show a demand change within the desired range, supporting the demand law, if prices are arbitrarily assigned. However, in Fig. 2.5, if the same prices are presumed, we cannot control the demand within the desired range of the Pareto–Slutsky equation.

2.2.2 Design of the Demand Law and a New Form

Abraham Wald was the first scholar to tackle the demand law in a scientific way. Let $p$ be price, and $f(p)$ be the demand at price $p$:

$$(p' - p'')(f(p') - f(p'')) \leq 0 \text{ i.e., } dp \cdot df \leq 0$$

(2.28)

This is the law of demand. We simply illustrate the case of two commodities. The demand column vector and the price row vector are denoted as:

$$p = (p_1, p_2)^{\text{transpose}}$$
Fig. 2.4 A normal case for the Pareto–Slutsky equation: \( \Delta p \Delta f < 0 \)

Fig. 2.5 An irregular case for the Pareto–Slutsky equation: \( \Delta p \Delta f \geq 0 \)
It then holds that:

\[ p_i' f(p_i') - p_i'' f(p_i'') \leq 0 \text{ for any } i \]
\[ \Rightarrow p_i' \leq p_i'' \rightarrow p_i' f(p_i') \geq p_i'' f(p_i'') \text{ for any } i \]  

(2.30)

Now suppose that:

\[ p' \neq p'', p_1' \geq p_1'', p_2' \leq p_2'' \]  

(2.31)

Then:

\[ p_1' \geq p_1'' \rightarrow p_1' f(p_1') \leq p_1'' f(p_1'') \text{ for any } 1 \]
\[ p_2' \leq p_2'' \rightarrow p_2' f(p_2') \geq p_2'' f(p_2'') \text{ for any } 2 \]  

(2.32)

The law describes the variation between a price \( p_i \) and its demand \( f(p_i) \). That is to say, a higher price corresponds to a lower demand in a strict form, and a non-decreasing price corresponds to a non-increasing demand in a weak form. This formulation is exempt from any specification of preference. Wald (1933) thus examined the mathematical property of \( f(p_i) \). This can be shown by geometry on two-dimensional coordinates (see Fig. 2.6).

### 2.2.3 A Numerical Derivation of a Demand Function

We can easily verify the complicated behavior of prices seen above, and a basic example of optimal household demand when we observe the barter exchange economy illustrated by an Edgeworth Box Diagram of two commodities, goods \( \{1, 2\} \) and two agents. The initial asset holdings \( (e_1, e_2) \) give the initial incomes of the agents. The income levels are represented by the relationship: \( p_1 e_1 + p_2 e_2 \). Using an individual utility function of the Cobb–Douglas type, \( u = \sqrt{x_1, x_2} \), we then set the utility maximization problem as:

\[
\max \sqrt{x_1, x_2} \text{ s.t. } p_1 x_1 + p_2 x_2 \leq p_1 e_1 + p_2 e_2
\]  

(2.33)
2.2 Analytical Examination of the Demand Law

By applying the Lagrangian method\textsuperscript{10} to this, we can reach the solution:

\begin{align*}
  x_1 &= \frac{p_1 e_1 + p_2 e_2}{2p_1} \\
  x_2 &= \frac{p_1 e_1 + p_2 e_2}{2p_2}
\end{align*}

(2.34) \quad (2.35)

It immediately follows that the demand for any good depends not only on its own price but also the price of alternatives and the initial asset holdings. It seems quite obvious that demand is always affected by price variations as well as the agent’s asset variations. The total differentiation will then give:

\[ dx_1 = \frac{e_2 (p_1 \Delta p_2 - p_2 \Delta p_2)}{2p_1^2} \]

(2.36)

2.2.4 The Demand Law as Solved by Hildenbrand (1994)

For simplicity, we suppose a one-commodity world, as proposed by Lewbel (1994). In his exposition, an essential feature of Hildenbrand’s solution is condensed, so we

\textsuperscript{10}In other words, this method is the method of Lagrangian multiplier to get a constrained extremum.
do not take into account relative prices. We saw in the Pareto–Slutsky equation of a one-commodity world that a supplemented item with income compensated to keep the same satisfaction is canceled out by the income effect:

$$\frac{\partial f}{\partial p} = \left( \frac{\partial f}{\partial p} + \frac{\partial f}{\partial x} \right) - \left( \frac{\partial f}{\partial x} f \right)$$  \hspace{1cm} (2.37)$$

However, a trivial transformation of this equation, replacing the first item with $s$ (the substitution effect), leads to:

$$\frac{\partial f}{\partial p} = s - \left( \frac{1}{2} \frac{\partial f^2}{\partial x} \right)$$  \hspace{1cm} (2.38)$$

### 2.2.4.1 A Sufficient Condition of the Demand Law in Hildenbrand (1994)

We introduce a population of heterogeneous consumers, so we have a distribution of individual demands $\{f_1, \ldots, f_i, \ldots, f_n\}$ constituted by individuals $i = 1, \ldots, n$. We omit the index $i$ for a while. We also define a new function $R(x)$:

$$R'(x) = \frac{\partial R(x)}{\partial x} = \frac{\partial f^2}{\partial x}$$  \hspace{1cm} (2.39)$$

It is clear from the above that:

$$R(x) = E(f^2)$$  \hspace{1cm} (2.40)$$

If we apply operator $E$ to a demand distribution, we will obtain a new formulation of the Pareto–Slutsky equation:

$$\frac{\partial E(f)}{\partial p} = E(s) - \frac{1}{2} E \left( \frac{\partial f^2}{\partial x} \right) = E(s) - \frac{1}{2} E[R'(x)]$$  \hspace{1cm} (2.41)$$

It follows from the property of the substitution effect that:

$$E(s) < 0$$  \hspace{1cm} (2.42)$$

Hence our new function $R(x)$ can be written:

$$R(x) = \{E(f^2) - E(f)^2\} + E(f)^2$$  \hspace{1cm} (2.43)$$

**variance** square mean of $f$  \hspace{1cm} (2.44)$$
2.3 Reconstructing Demand Theory

We can therefore measure the spread of demand by \( R(x) \). Demand, as well as heterogeneity of households, is greater as \( R(x) \) increases. Suppose that \( \rho \) is a density function of income \( x \). It then holds that:

\[
E[R'(x)] = \int R'(x)\rho(x)\,dx = \int E\left(\frac{\partial f^2}{\partial x}\right)\rho(x)\,dx > 0 \quad (2.45)
\]

It is therefore always certain, because of the property of \( R(x) \), that \( x \) increases.

\[
\frac{\partial E(f)}{\partial p} < 0 \quad (2.46)
\]

Again, this is the demand law. It is verified by a sufficient condition that the spread of demand \( R(x) \) is greater as income \( x \) increases.

2.2.4.2 The Significance of Hildenbrand’s Finding

In traditional economics, price fluctuations overwhelmingly dominate changes in household demand almost everywhere. The demand law is distorted by the income effect. We have already shown that the direction of household demand is never driven only by price changes. Because of Hildenbrand’s (1994) contribution, we also know that class structure works to create a normal form of the demand law. It is quite interesting that a seemingly ‘normal’ law can actually be realized by supplementing a social factor: the existence of heterogeneous agents.

This perspective may be diagrammatically depicted (see Fig. 2.7). First, we recognized that income effects are not auxiliary, even where price-sensitive behavior is dominant, as long as prices appear independent, hence there are different forces: prices, demand and income classes. We can then show the real facets of interacting prices and demands with different income classes. As Fig. 2.7 shows, we need a multi-dimensional view. Our vision depends on which facet we view. The observation of consumer demand should not be limited to a particular facet where price-sensitive behavior could be dominant if prices are largely independent.

2.3 Reconstructing Demand Theory

By introducing the different income classes in the previous section, we clarified correlations between heterogeneous consumers. In this section, we will consider forces other than the contribution from these agents. Such an attempt may help us explore several hidden forces that shape household demand. The demand law may also be affected by other forces such as nonrandom modes reflecting natural or social necessities, and business fluctuations. We may therefore conceive that another force, distinct from individual inner preferences, is at work. This then
suggests another problem with the demand law. By applying a new method like random matrix theory to empirical household data, a remarkable feature has been found that is common to different class behaviors, irrespective of price variations. A resulting consumption activity may not be decided solely by price and income. There is another independent force driving consumption, which may be estimated by the system itself; in other words, the eigenmodes of the system. It is evident that consumer behavior is closely connected to certain social patterns.

2.3.1 Self-organizing Patterns of Consumption

So far, we have confined the objects of analysis to a closed domain, to permit a mathematically complete analysis. Once the domain equipped with the desired properties is built up, the analysis will succeed, providing we have sufficient mathematical skill. Here “domain” may be interpreted as “environment”. In this sense, traditional economic thinking may be classified as “environmentalism”. However, in reality, human satisfaction and consumption cannot be confined to a particular closed space. The space is always exposed to the external environment and its fluctuations. The resulting consumption pattern may be considered as a self-organizing activity affected by the outer world. The idea of mathematical closedness both in the commodity space and in terms of personal satisfaction must be a fallacy, if we envisage the way in which patterns of consumption form in reality. Outside sophisticated mathematical modeling, we need a different idea of consumption.
2.3 Reconstructing Demand Theory

2.3.1.1 Formation of Patterns of Consumption

The formation of patterns of consumption is shown in Fig. 2.8. Market field is the domain supposed by traditional economics, a hypothetical reference. The link between price and demand is intermediated by the substitution and income effects. The income effect is no longer fixed outside different income classes, known as “the socioeconomic system”. An actual pattern of consumption will appear in the range of two domains, i.e., the socioeconomic system. This section sets out the formation of the pattern of consumption.

We can easily obtain the yearly income of all households of five income rank classifications, with monthly receipts and disbursements per household, for instance. Table 2.2 lists ten categories of expenses, and there are five income ranks. If there are any similar major principal modes, each income class can be derived from empirical data by a spectra distribution, and also be separated explicitly from the derived random distribution by the random matrix theory. It can therefore be verified that there is a common factor over the different income classes, and the effects of macroeconomic variables on the spending categories over the different income classes can be identified.

In reality, the members belonging to an income class are constantly changing. A member may fall into a lower income class, or rise to the next one. We assume that the movements between classes cancel each other out because of a limitation.

---

11 See Monthly Expenditure per Household (MEH) in Statistics Bureau, Ministry of Internal Affairs and Communications of Japan (2013a). The Report on Family Income and Expenditure in Statistics Bureau, Ministry of Internal Affairs and Communications of Japan (2013b) sets out ten categories with five income classes. The Indices of Industrial Production (IIP) in Japan provide 21 categories of goods with three ranks of production (value added), shipment, and inventory.
Table 2.2 The categories of expense items

<table>
<thead>
<tr>
<th>Expense items</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Food</td>
</tr>
<tr>
<td>2. Housing</td>
</tr>
<tr>
<td>3. Fuel, light, and water charges</td>
</tr>
<tr>
<td>4. Furniture and household utensils</td>
</tr>
<tr>
<td>5. Clothing and footwear</td>
</tr>
<tr>
<td>6. Medical care</td>
</tr>
<tr>
<td>7. Transportation and communication</td>
</tr>
<tr>
<td>8. Education</td>
</tr>
<tr>
<td>9. Culture and recreation</td>
</tr>
<tr>
<td>10. Other</td>
</tr>
</tbody>
</table>

Table 2.3 Mode 1 (eigenvalue $\lambda_1 = 4.24$)

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Food</td>
<td>0.288</td>
<td>0.275</td>
<td>0.281</td>
<td>0.283</td>
<td>0.335</td>
</tr>
<tr>
<td>2 Housing</td>
<td>0.069</td>
<td>0.070</td>
<td>0.024</td>
<td>0.034</td>
<td>0.018</td>
</tr>
<tr>
<td>3 Fuel, light and water charges</td>
<td>0.088</td>
<td>0.193</td>
<td>0.189</td>
<td>0.161</td>
<td>0.128</td>
</tr>
<tr>
<td>4 Furniture and household utensils</td>
<td>0.137</td>
<td>0.091</td>
<td>0.067</td>
<td>0.153</td>
<td>0.030</td>
</tr>
<tr>
<td>5 Clothing and footwear</td>
<td>0.086</td>
<td>0.264</td>
<td>0.058</td>
<td>0.161</td>
<td>0.176</td>
</tr>
<tr>
<td>6 Medical care</td>
<td>0.049</td>
<td>0.059</td>
<td>0.115</td>
<td>0.109</td>
<td>0.032</td>
</tr>
<tr>
<td>7 Transportation and communication</td>
<td>0.098</td>
<td>0.083</td>
<td>-0.056</td>
<td>0.169</td>
<td>0.005</td>
</tr>
<tr>
<td>8 Education</td>
<td>0.005</td>
<td>0.088</td>
<td>0.035</td>
<td>-0.160</td>
<td>0.071</td>
</tr>
<tr>
<td>9 Culture and recreation</td>
<td>0.141</td>
<td>0.171</td>
<td>0.071</td>
<td>0.043</td>
<td>0.171</td>
</tr>
<tr>
<td>10 Other</td>
<td>-0.056</td>
<td>0.096</td>
<td>0.094</td>
<td>-0.019</td>
<td>0.122</td>
</tr>
</tbody>
</table>

Note: The data used are from Jan 2000 to Feb 2012

of statistical data. Types are then replaced with state variables. Hildenbrand (1994) called this assumption metonymy.

The sample numeric space is therefore given by five income classes $\times 10$ household expense items $\times$ the number of months. By deriving the correlation matrix, we can calculate its eigenmodes and which components are constituted by the items of consumption. Each income class has its own eigenmode, as Table 2.3 shows. We may then call the eigenmode “the pattern of consumption”. By comparing the eigenvector component sizes corresponding to the expense items in this mode, and common to different classes, we see that FOOD is the dominant factor. This mode could then be called the FOOD-dominant mode. The FOOD item is paired with the remaining items. Spending on these factors may move with FOOD spending as the consumption level varies. This mode indicates a pattern of consumption around FOOD.

In the next subsection, we will see an independent force separated from a supposed market field. A new tool can be used to identify a nonrandom eigenmode among those of a socioeconomic system, and across the whole period under
2.3 Reconstructing Demand Theory

investigation. We will apply random matrix theory to sample data to empirically verify that some eigenmodes could be judged as nonrandom independently of a hypothetical market field.

2.3.1.2 A Force Common to All Income Classes

Before checking the empirical properties of patterns of consumption in Japan, it may be helpful to sum up two distinct forces:

1. A force to cancel out irregularities in demand direction: class differences may contribute to mitigate irregularities, as Hildenbrand (1994) noted (see Sect. 2.2).
2. A force to create a self-organizing assimilation of a segmented consumption basket, whether partly class-specific or common to all classes. Some independent forces may work the same way across all classes rather than differently for different classes.

2.3.2 An Empirical Analysis of Patterns of Consumption

In standard economics textbooks, there is a typical taxonomy of substitutability between coffee and tea, as well as complementarity between coffee and milk. However, such a taxonomy may give us only a superficial classification of price behaviors, because we are less able to decide which good is which by any empirical token. In the case of complementarity, by definition, an expected behavior merely suggests that demand for each element of a complementary pair has a common direction. But the inverse relation may not hold because of the income effect.

Figure 2.8 shows a self-organizing pattern of consumption. We will now move away from the classical inference in the theory of demand. In the classical school, the choice space has no room for a self-organizational force to work. A pattern of consumption is a product of systemic creation. Such a pattern of consumption cannot penetrate the choice space, because this may be partially distorted if choices are largely driven by price variations. In our instance, each income class is characterized by an eigenmode. The free choice set is always about to be encompassed by a self-organizing force emerging from interacting human minds and socio-customs. In some cases, such a force may secure its own direction.

2.3.2.1 An Alternative Way to Affect Household Demand

We now focus on the factors that work together to cause consumer demand fluctuations. There are two main ways to affect household demand:

1. The income effect
2. Other effects generated by some correlative links (nonrandom factors) either as a result of the necessities of life or some macroscopic business fluctuations.
We use the Japanese economy to monitor the effects of macroscopic correlations among consumption categories of different income classes or heterogeneous agents, and then identify the other nonrandom effects, if any, due to business fluctuations. These ideas depend on Iyetomi’s idea (Iyetomi et al. 2011a, b) of distinguishing the principal modes at different industrial levels to construct macroscopic effective demands from random modes using the principle of the random matrix. Iyetomi et al. (2011a, b) showed that there is a structural (nonrandom) cycle in the Japanese economy, and judged the existence of the business cycle by applying the random matrix theory (RMT) to the empirical data of the Indices of Industrial Production (IIP) in Japan.

We follow Iyetomi et al. (2011a, b) in using industrial data. The variable is \( S_{\alpha,g}(t_j) \), where \( \alpha \) denotes an income class, going from one at the bottom to five at the top, \( g \) denotes the ten household expense categories, and \( t_j \) is expressed by \( t_j = j \Delta t \). Here \( \Delta t = 1 \) is a month, and \( j \) runs from 1 to \( N \) (\( N = 146 \) for Jan 2001 to Feb 2012). It then holds for the logarithmic growth rate that:

\[
r_{\alpha,g}(t_j) := \log_{10} \frac{S_{\alpha,g}(t_{j+1})}{S_{\alpha,g}(t_j)}
\]

(2.47)

We then normalize them:

\[
w_{\alpha,g} := \frac{\sigma_{\alpha,g}(t_j) - <r_{\alpha,g}>_t}{\sigma_{\alpha,g}}
\]

(2.48)

Here \(<r_{\alpha,g}>_t \) is the average over all \( t_j \), and \( \sigma_{\alpha,g} \) is the standard deviation of \( r_{\alpha,g} \) over time. The average is zero, and the standard deviation is one, making it suitable for random matrix theory application.

Next, we apply the Fourier decomposition to \( w_{\alpha,g}(t_j) \):

\[
w_{\alpha,g} = \frac{1}{\sqrt{N'}} \sum_{k=0}^{N'-1} \tilde{w}_{\alpha,g}(\omega_k) e^{i\omega_k t_j}
\]

(2.49)

The frequency is defined as:

\[
\omega_k = \frac{2\pi k}{N' \Delta t}
\]

(2.50)

Since the figures are all real values, it follows that:

\[
\tilde{\omega}^*_{\alpha,g}(\omega_k) = \tilde{\omega}_{\alpha,g}(\omega_{N'-k})
\]

(2.51)

It then also follows that:

\[
\tilde{\omega}_{\alpha,g}(0) = 0
\]

\[
\sum_{k=0}^{N'-1} |\tilde{\omega}_{\alpha,g}(\omega_k)|^2 = N'
\]

(2.52)
Thus we can define the average power spectrum $p(\omega_k)$ as:

$$p(\omega_k) = \frac{1}{M} \sum_{\alpha=1}^{5} \sum_{g=1}^{10} |\tilde{w}_{\alpha,g} (\omega_k)|^2$$  \hspace{1cm} (2.53)

In our case, $M = 50$. It also follows that:

$$\sum_{k=0}^{N'-1} p(\omega_k) = N'$$  \hspace{1cm} (2.54)

We define the correlation matrix as $C$. It is composed of elements $w_{\alpha,g}$, $w_{\beta,l}$ whose diagonal is 1, because of the definition of $w_{\alpha,g}$.

$$C_{\alpha,g,\beta,l} = \langle w_{\alpha,g} w_{\beta,l} \rangle_t$$  \hspace{1cm} (2.55)

$\alpha$ and $\beta$ run from one to five, and $g$ and $l$ from one to ten. The eigenvectors $V(n)$ and eigenvalues $\lambda(n)$ are associated with this coefficient matrix $C$, which is then expressed as:

$$C = \sum_{n=1}^{M} \lambda^{(n)} V^{(n)} V^{(n)\text{transpose}}$$  \hspace{1cm} (2.56)

If the data are not all random in the space, we can usually find a distribution of eigenvalues with this configuration, with a few isolated large eigenvalues explicitly separated from the range of smaller values. The largest group of eigenvalues and associated eigenvectors then corresponds to the dominant factors, which explains co-movements of the fluctuations of different goods. We consider the largest eigenvector to be the first. This vector can be interpreted as the first major or principal component, which accounts for the most influential combination of correlated factors (different goods) over the space of $M$-dimension. Similarly, the eigenvector for the second-largest eigenvalue is the second principal component that accounts for as much of the correlation as possible in the subspace orthogonal to the first eigenvector, and so on (Iyetomi et al. 2011a). In other words, the second component has an influence independent of the first.
formulated previously, we obtained the distributions in Figs. 2.9 and 2.10 (the eigenvalue distribution of household consumption in Japan for Jan 2000 to Feb 2012).

We used a computer program to make a seasonal adjustment to the data, and more detail about this is provided later. We also produced the two largest modes of correlated components over different income classes: Figs. 2.11 and 2.12 (Tables 2.4 and 2.5).

2.4.2 Comparison Between Alternative Seasonal Adjustments

When we deal with consumer data in general, we need to make seasonal adjustments. We used two main alternative methods of adjustments on our data. The first
2.4 The Results of Statistical Verification

Fig. 2.11 The eigenvalue distribution of mode 2

Fig. 2.12 Comparison between X-12-ARIMA and DECOMP
Table 2.4  Mode 1 (eigenvalue $\lambda_1 = 4.24$)

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.288</td>
<td>0.275</td>
<td>0.281</td>
<td>0.283</td>
<td>0.335</td>
</tr>
<tr>
<td>Housing</td>
<td>-0.069</td>
<td>-0.070</td>
<td>-0.024</td>
<td>0.034</td>
<td>0.018</td>
</tr>
<tr>
<td>Fuel, light and water charges</td>
<td>0.088</td>
<td>0.193</td>
<td>0.189</td>
<td>0.161</td>
<td>0.128</td>
</tr>
<tr>
<td>Furniture and household utensils</td>
<td>0.137</td>
<td>0.091</td>
<td>0.067</td>
<td>0.153</td>
<td>0.030</td>
</tr>
<tr>
<td>Clothing and footwear</td>
<td>0.086</td>
<td>0.264</td>
<td>0.058</td>
<td>0.161</td>
<td>0.176</td>
</tr>
<tr>
<td>Medical care</td>
<td>0.049</td>
<td>-0.059</td>
<td>0.115</td>
<td>0.109</td>
<td>-0.032</td>
</tr>
<tr>
<td>Transportation and communication</td>
<td>0.098</td>
<td>0.083</td>
<td>-0.056</td>
<td>0.169</td>
<td>0.005</td>
</tr>
<tr>
<td>Education</td>
<td>0.005</td>
<td>-0.088</td>
<td>0.035</td>
<td>-0.160</td>
<td>0.071</td>
</tr>
<tr>
<td>Culture and recreation</td>
<td>0.141</td>
<td>0.171</td>
<td>0.071</td>
<td>0.043</td>
<td>0.171</td>
</tr>
<tr>
<td>Other</td>
<td>-0.056</td>
<td>0.096</td>
<td>0.094</td>
<td>-0.019</td>
<td>0.122</td>
</tr>
</tbody>
</table>

Note: The data used are from Jan 2000 to Feb 2012

Table 2.5  Mode 2 (eigenvalues $\lambda_2 = 3.54$)

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>-0.073</td>
<td>-0.009</td>
<td>-0.171</td>
<td>-0.104</td>
<td>-0.108</td>
</tr>
<tr>
<td>Housing</td>
<td>0.002</td>
<td>0.076</td>
<td>0.139</td>
<td>-0.023</td>
<td>0.079</td>
</tr>
<tr>
<td>Fuel, light and water charges</td>
<td>0.362</td>
<td>0.327</td>
<td>0.310</td>
<td>0.342</td>
<td>0.355</td>
</tr>
<tr>
<td>Furniture and household utensils</td>
<td>0.000</td>
<td>0.065</td>
<td>-0.058</td>
<td>-0.106</td>
<td>0.061</td>
</tr>
<tr>
<td>Clothing and footwear</td>
<td>-0.064</td>
<td>-0.027</td>
<td>-0.133</td>
<td>-0.210</td>
<td>-0.081</td>
</tr>
<tr>
<td>Medical care</td>
<td>-0.175</td>
<td>0.102</td>
<td>-0.073</td>
<td>-0.046</td>
<td>0.121</td>
</tr>
<tr>
<td>Transportation and communication</td>
<td>-0.025</td>
<td>0.059</td>
<td>0.021</td>
<td>-0.028</td>
<td>0.137</td>
</tr>
<tr>
<td>Education</td>
<td>0.001</td>
<td>0.143</td>
<td>0.012</td>
<td>0.010</td>
<td>0.101</td>
</tr>
<tr>
<td>Culture and recreation</td>
<td>0.217</td>
<td>0.010</td>
<td>-0.028</td>
<td>0.089</td>
<td>-0.026</td>
</tr>
<tr>
<td>Other</td>
<td>-0.162</td>
<td>-0.002</td>
<td>-0.141</td>
<td>-0.072</td>
<td>-0.111</td>
</tr>
</tbody>
</table>

Note: The data used are from Jan 2000 to Feb 2012

was X-12-ARIMA,\textsuperscript{12} developed and used by the U.S. Census Bureau. It is also a standard seasonal adjustment method used by the Statistics Bureau in Japan. The X-12-ARIMA program draws on experimental evidence, with a number of degrees of freedom for users to optimize the procedure. However, we allowed the program to determine a best-fit model by itself. The second was DECOMP,\textsuperscript{13} developed by the Institute of Statistical Mathematics, Japan. The basis of the two is quite different. In our application, surprisingly, the derived results are very similar for the principal components, and show that the largest eigenvalue is dominated by FOOD.

\textsuperscript{12}United States Census Bureau (2013). ARIMA is the acronym of Auto-Regressive Integrated Moving Average.

\textsuperscript{13}DECOMP (Kitagawa and Gersch 1984) decomposes a given time series data into trend, seasonal and irregular components in a transparent way. It is based on state-space-modeling. It is therefore free from the moving average procedure that plays an important role in X-12-ARIMA. However, we did not have much room to play with the program to optimize the procedure.
and the second by FUEL, LIGHT, & WATER. In other words, the first component is constructed by the explanatory category dominated by FOOD, and the second by the category dominated by FUEL, LIGHT & WATER. These results are shown diagrammatically in Fig. 2.12.

2.5 Some Implications Derived from Statistical Tests

2.5.1 Main Findings

We can derive several implications from our statistical observations. We have found two statistically significant principal modes distinguished from a random distribution of eigenvalues. In particular, we have found modes 1 and 2, associated with the eigenvalues $\lambda_1 > \lambda_2$.

The first principal mode is FOOD. It turns out that mode 1 may contribute 8.5% to the total variation in consumption. The second principal mode is FUEL, LIGHT & WATER. This contributes up to 7.1% of the total variation in consumption. The result is similar, irrespective of the seasonal adjustment method used. We thus conclude that the FOOD-dominant and FUEL-dominant modes are the driving forces for the pattern of consumption.

We then turn to the sign of the components. The signs of the FOOD and FUEL components are positive in mode 1, but opposite in mode 2, as seen in Fig. 2.11.

The result of mode 1 seems obvious, if we interpret the FOOD mode in the context of Engel’s coefficient connections. If we cook FOOD items, we will also use FUEL. In mode 2, however, it is hard to find an obvious interpretation. CLOTHING has negative signs in all the different income classes. In mode 2, then, the FUEL consumption is opposite to that of FOOD as well as CLOTHING. We therefore suggest that mode 2 might be close to the consumption activity of requiring more electricity, for example, by using a personal computer. In the event, whatever interpretation is given, we have shown that about 15% of the total variation in consumption is generated by an organized co-movement in a particular way, whether natural or cultural. We cannot, however, know whether the remaining distribution is genuinely random in view of the random matrix theory.

To detect any correlation induced by hidden factors, we can investigate a possible relationship. We look for mode 3($\lambda_1 > \lambda_2 > \lambda_3$). A mode with eigenvalues below 0.1 may be regarded as completely random. However, if mode 3 has a larger standard deviation than the random variable, this could be from another factor. Alternatively, we could add three random variables. A tentative principal analysis in the expanded data showed little difference from the previous analyses, although another calculation indicated that a mode around the added variables showed some substantial changes in standard deviation. This may imply a visible correlation with business fluctuation, although this remains unverified.
2.5.2 Further Findings

We made another attempt to detect hidden factors affecting consumer demands, choosing two factors, Extra Working Hours and TEMPERATURE. Extra working hours may be sensitive to business fluctuations. Our results were:

1. After we added the time series of extra working hours to the original data, we calculated the eigenvector distributions. The extra working hours as an additional 11th element marked a positive high value, particularly in mode 1, but also in mode 2.

2. We added the anomaly time series of monthly average temperatures in Tokyo Metropolitan City, and found that the pattern was unchanged for both modes, although the components of mode 2, in particular, were magnified by temperature.

Figure 2.13 shows, on the horizontal axis, the column before last that corresponds to Extra Working Hours, and the last that corresponds to Temperatures.\(^\text{14}\)

\(^{14}\)We used the database in a year-on-year basis.
The first principal eigenvector retains the positive correlation over all components. Once the time series of Extra Working Hours was added, FOOD, FUEL and Extra Working Hours made a large contribution to the first principal eigenvector. We may therefore regard Extra Working Hours as an important parameter for business fluctuations, and mode 1 probably depends on business fluctuations. In mode 2, FUEL and CLOTHING are closely correlated with TEMPERATURE, so the consumption pattern of mode 2 may be most sensitive to TEMPERATURE.

Most of the items of expenditure have strong seasonal dependence, as expected, except for Housing, Medical and Transport. Removing seasonal components is therefore critical to elucidate possible correlations in expenditure by Japanese consumers.

References


Wald A (1933–34) Über die Eindeutig Positive Löbarkeit der Neuen Produktionsgleichungen. Ergebnisse eines Mathematischen Kolloquiums 6:12–20
Chapter 3  
Network Analysis of Production and Its Renewal

Abstract  Without positioning a production system, it is impossible to imagine any economic doctrine. During recent years, however, many economists have been very reluctant to use actual observations on the structure of production systems and heterogeneous interactions among production processes, i.e., inter-industrial analysis, and commit to a positive picture of production in society. Instead, they have focused on a particular idea of the production function, where the aggregate level is very similar to the microscopic one. **The representative agent** is ubiquitous in standard economics, across the globe and historically. As Robert C. Allen (2011) noted, some economists do not hesitate to subordinate even human history into a special production function. They stress behavioral decisions by game theoretic analysis to dispense with productive structural analysis. In this chapter, we will examine the meaning of a price system based on the production system in the sense of classical economic doctrines, and will then give a new network analysis of production and consider its empirical application to the Japanese economy over recent years.

3.1 Changes in the Concept of Price over the Last Century

By the 1970s, the keen interest in the theory of price and production that had dominated the 1920s and 1930s had decreased drastically. The contributions of von Neumann, Wald, Remak, von Cassel, von Stackelberg, Schneider, and Samuelson, among others, motivated many scientists to study economics, which led to the growth of mathematical economics in the mid-1950s. The existence and stability proofs of general equilibrium were therefore achieved with remarkable mathematical precision. However, by the 1970s, many of the books on production and distribution were no longer on sale, demonstrating the metamorphosis of fundamental business activities in advanced countries as the banking business and other advanced industries began to computerize their activities.

Here, imperfect or irrational conditions no longer matter. More significantly, equilibrium prices no longer play a primary role in realizing products in the market. The present mode of production is subject to increasing returns and to the market generating a path-dependent process of trading shares, as Arthur (1994) demonstrated. This process is accompanied by a persistent innovation process,
bringing steady change to the economic environment. In this mode of production, the novelty of products becomes more important. This feature is quite similar to biological evolution, as Holland (1992, 1995) noted in his genetic algorithm. In other words, a new mode of production emerged, starting at the end of the last century, which in turn changed the bidding system. The economic system can operate without resorting to equilibrium prices; producers need not employ the equilibrium price system to survive. The agents in an economy always receive feedback from other agents’ reactions. In an actual system accompanying persistent innovations, the environment should always be renormalized.

### 3.1.1 Shift in Trading Methods and the Environmental Niche

In short, price no longer plays a crucial role in the economy. The equilibrium price system is no longer the sole decisive factor in the fulfillment of trades. While price is still a principal source of information, it is not the final determinant in equilibrating supply and demand. The prices actually employed are no longer the so-called equilibrium prices that eliminate excess demand. This alteration suggests that a replacement of the trading method will occur. In fact, the amount traded through “batch auctions” has shrunk drastically, while “continuous double auctions” have become much more dominant. As illustrated in Chap. 4, batch auction depends on price priority, while double auction depends on time priority. In the latter, the settlement is attained by matching ask and bid, according to the time preference principle. This observation is not limited to the stock exchange, but may also be more generally applied to the process of selecting a productive plan. This shift is a reflection of the metamorphosis of production. General equilibrium prices are no longer required. The bid/ask mechanism for selecting an adaptive productive plan might be replaced by the bucket brigade algorithm as an internal mechanism, as the evolutionary genetic algorithm shows. The algorithm has been developed by the idea of “complex adaptive system” originally defined by Holland (1995, Chap. 1). The evolutionary process based on this algorithm moves to a new stage via the feedback loop from a consecutively modified environment. The genetic algorithm will be explained further in Sect. 3.2 of this chapter. See Fig. 3.2.

### 3.1.2 Classical Steps Towards Equilibrium

As Aruka and Koyama (2011, p. 149) demonstrated, the agent who sets the desired amount of a set of \{price, quantity\} to trade with an arbitrary agent must have an independent plan, which is never influenced by the external environment. Consequently, the agent must cancel the original offer if things do not go according to plan. Suppose that the agent is a buyer who wants to purchase a desired good at a price slightly higher than the asking price. In this case, a rule will be applied:
1. In the case that there is a market order to sell, any purchase plan could be at least partly fulfilled. Here, let the concerned buyer be the person who obtains a contract.
2. If the asking price rises to a price higher than planned, the buyer must then give up the contract right to purchase.
3. The buyer must then become a seller of the obtained amount, adding to the market supply. However, this transition from buyer to seller will be not realized in the Walrasian tâtonnement, because all the contracts obtained before equilibrium are canceled.

3.1.3 Application of a Genetic Algorithm to the Economic System

In the classifier system set out by Holland (1992, Chap. 10; 1995, Chap. 2), genetic evolution will be achieved by the following mechanisms:

1. The learning process to find a rule
2. The credit assignment on rules
3. Rule discovery

The learning process has an interactive mechanism involving the defector (on input), effector (on output), and the environment. The key issues for motivating an evolutionary system are stage-setting and adaption of the rule to the environment, which are taken into consideration by the bucket brigade algorithm. Decisions (or intentions) in an economic system might be regarded as stage-setting moves (Holland 1992, p. 177).

In such a feedback system, agents cannot necessarily immediately identify equilibrium prices. The environment in which the agents operate will be changed before equilibrium has been detected. Practically, then, equilibrium prices might be replaced by quasi-mean values in a dynamic process of production and trading. In parallel with this replacement, the bidding process for production costs might also be changed. A feedback system will therefore change the world of equilibrium into a true evolutionary economic process.

In a genetic algorithm, a gene can evolve through bidding or asking a price, provided its bid or price matches a receiver’s or consumer’s offer. In this trade, a kind of double auction is at work. However, there is no auctioneer or neutral mediator in the bidding, so the participants refer only to expected mean advantages or the quasi-mean fitness of a particular ensemble of genes. Suppliers might use such a mean value as the basis for asking prices.

We now turn to the von Neumann economic system in light of Holland (1992, 1995). Holland ingeniously envisaged that the von Neumann economic system could be reformulated as a genetic algorithm, although von Neumann’s original bid/ask mechanism differs from new systems such as a bucket brigade algorithm.
Here, emphasis is placed on the adaptive capability of a process in its environment. This chapter will focus on the internal selection mechanism of the original von Neumann balanced economic system, in light of a genetic algorithm. This argument may also provide a new insight into the Sraffa standard commodity, which is also found in the original von Neumann model.

### 3.1.4 Significance of the Standard Commodity, in the Context of the Genetic Algorithm

#### 3.1.4.1 The Classical Meaning of Prices

Despite the work of Sraffa (1960), the significance of the “standard commodity” has still not been elucidated satisfactorily. One of the major reasons for its ambiguity may be the impossibility of it occurring in a general case such as a joint-production system; for example, the von Neumann economic system. This view can easily lead to the devaluation of its contribution, especially since there is no proper equilibrium in a more general case. Sraffa’s original concern, however, was irrelevant to the price equilibrium. The classical principle of political economy refers to the natural price, or prices other than the equilibrium prices, as the center of gravity in the economic system. It is therefore clear that the classical meaning of prices is not the same as prices in general equilibrium.

#### 3.1.4.2 Property of the Original von Neumann Solution Based on the Constancy of the Environment

The solution of the von Neumann economic system contains optimal prices and quantities. It is derived by referring to the payoff function (the value of the game). Von Neumann employed the two-person strategy game of growth-maximizing and cost-minimizing players to prove the solution and establish equilibrium. However, it is difficult to interpret his original proof with an actual counterpart because, as previously pointed out, both players are aggregates and so hypothetical agents. In this construction, equilibrium, as a quasi-stationary state, only attains optimality over time. The balanced growth or sustainability therefore simply requires constancy in its environment. Once the equilibrium has been attained, the state should be constant forever. However, the internal selection mechanism is missing: the exchange of information with the environment in the original von Neumann system. As the environment changes before the end of the time horizon on a productive plan, equilibrium cannot help being changed or revised. The constancy of the environment must be relaxed, which will be accompanied by a feedback system, as in the bucket brigade algorithm. This algorithm can run a “classifier system” as shown in Sect. 3.2.5. See Fig. 3.2.
3.1.4.3 Introduction of a Genetic Algorithm into the Original Sraffa System

The same limitation seems to be found in Sraffa’s system too. However, Sraffa’s original intention was to measure fluctuating price movements in terms of the invariant measure of the value of the standard commodity. Any idea of production scale was not presupposed by his essential image of the production of commodities by means of commodities, so his system did not need constancy of environment. The standard commodity in the original sense is at only one point of production. In other words, the standard commodity simply gives a momentary profile of the system. As time goes by, it must change and be continuously updated. If we extend our vision to the future evolution of the economic system, it is clear that the standard commodity must evolve.

The standard commodity represents a reference point for the sustainability of the system. The process of selection among productive plans requires an anchoring point by which the process will be evaluated. We must therefore look for a basis for calculating the mean value of the profitability of the economic system. However, actual systems do not always provide a square matrix and unique maximal vector. Therefore, some approximation is needed to find an equivalent surrogate system.

3.1.4.4 Derivation of the Standard Commodity

Linearity of the production process is not a requirement for derivation of the standard commodity. However, the assumption of linearity is convenient for proving the standard commodity, as the special commodity is the same idea as the non-negative eigenvector of the input matrix in the linear form of the production system. We assume the linearity of the production system. However, labor, $l$, as the sole primary factor of production, may be explicitly treated as a limiting factor in the Sraffa production system, which can take into account the distributive effects on the price system. We denote such an extended production system by $[A, B, l]$.

An exact description of the existence proof of the standard commodity is set out in Aruka (1991, 2012) and elsewhere. Here, we only show the significance of the standard commodity in the simplest production system. The preparation for this proof is:

Consider a linear production system without joint production, where each process produces a single product. Labor is the sole primary factor of production and is essential for each process. A is an $n \times n$ square matrix of input coefficients $a_{ij}$, $a_0$ is a labor input vector, $p$ is a price vector and $r$ is the rate of profit. $I$ is a unit matrix, with all elements zero except for 1 on the (continued)
diagonal. The simplest price system will then be an inhomogeneous equation system:

\[
[I - (1 + r)A]p = a^0
\]  

(3.1)

Here, the price of labor (the wage rate) is fixed at unity, and prices of produced goods are therefore expressed in terms of labor. By virtue of the following assumptions, we can obtain, as a solution, a non-negative vector:

1. An input matrix \( A \) is productive: \( q[I - A] \geq 0 \) for any \( q \geq 0 \).
2. An input matrix \( A \) as a whole is non-negative and irreducible, and its eigenvalue is smaller than \( 1/(1 + r) \).

Either guarantees a non-negative solution:

\[
[I - (1 + r)A]p = a^0
\]  

(3.2)

(2) confirms a unique non-negative solution. We can then prove the transformation of the Sraffa price system in view of the commutable matrices; that is, \([I - (1 + r)A]^{-1}A\) and \(A\) commutable:

\[
[I - (1 + r)A]^{-1}A \cdot A = A \cdot [I - (1 + r)A]^{-1} \text{ for } r \leq R. \]  

(3.3)

**Theorem 3.1.** Sraffa’s theorem (Aruka 1991, 2011, Chap. 2; Schefold 1976a,b) (i) Suppose \( sa^0 = 0 \) for \( s \neq 0 \), any left-hand eigenvector of matrix \( A \) (or \( a^0 \neq (1 + R)Aa^0 \)). If and only if \( a^0, Aa^0, A^2a^0, \ldots, A^{r-1}a^0 \) are linearly independent does there then exist a \( p \) such that:

\[
Ap = \frac{m}{1 + m(1 + r)} p \text{ for non-zero number } m. \]  

(3.4)

There also then exists a left-hand side eigenvector \( s \) such that:

\[
sA = \frac{m}{1 + m(1 + r)} p \text{ for non-zero number } m. \]  

(3.5)

Here, \( s \) is the standard commodity. (ii) Normal modes of the Sraffa price system are equivalent to the necessary and sufficient condition (i) of this theorem.

3.1.4.5 The Balancing Proportion

Sraffa originally demonstrated “the balancing proportion” for prices against the changes of a distributive variable. In the present framework of the production system, the balancing proportion may be represented by the following expression:
Let $m$ be the ratio satisfying the condition:

$$m = \frac{a_i p}{a_i^0} = \frac{a_i p}{a_i^0} = a_i p = \cdots \text{for } r \in (0, R].$$

(3.6)

The ratio $m$ is the balancing proportion for prices against the changes of the rate of profit. The activity vector $s$ preserving this critical proportion must satisfy the following condition:

$$m = \frac{s A^n p}{a_i A^{n-1} a^0} \text{ for } r \in (0, R].$$

(3.7)

It is therefore easy to ascertain that there is a unique non-negative eigenvector to support the balancing proportion $m$.\(^1\)

### 3.1.4.6 The Standard Commodity in the Sraffa Joint-Production System

We can insert the von Neumann system into the Sraffa production system as a special case. According to Gale (1956, p. 286), this kind of model is obtained by adding two extra conditions to the von Neumann model in von Neumann (1937, 1945–1946):

1. The number of basic production processes is the same as the number of basic goods.
2. Only one good is produced by each process.

We also define the Sraffa joint-production system $[A, B, a^0]$. Here, input and output matrices are of square forms. The price system of the joint-production system will be:

$$Bp = (1 + r)Ap + wa^0.$$  

(3.8)

If $[B - (1 + r)A]^{-1}$ exists, the solution will be given in the form:

$$p = [B - (1 + r)A]^{-1}a^0.$$  

(3.9)

\(^1\)See Aruka (1991, 2012). The precise mathematical derivation of this balancing proportion was firstly given by Schefold (1976a,b).
Here, \( w = 1 \). We then introduce the assumption that \( [B - A] \) can be invertible, so the solution could be transformed into:

\[
p = [I - r[B - A]^{-1}A]^{-1}[B - A]^{-1}a^0. \tag{3.10}
\]

The form of the solution reflects the idea of a vertically integrated system in which joint production can be reduced to a virtual single production system. The price system of a vertically integrated system is:

\[
[[B - A] - 1A, I, [B - A] - 1a0]. \tag{3.11}
\]

\( [B - A]^{-1}A \) may be regarded as input, \( I \) as output, and \( [B - A]^{-1}a^0 \) as labor \( l \) in an integrated system. For convenience, we denote this as:

\[
Z = [B - A]^{-1}A; l = [B - A]^{-1}a^0 \tag{3.12}
\]

It is easy to find the non-negative eigenvector \( h \geq 0 \) if \( Z = [B - A]^{-1}A \geq 0 \):

\[
hZ = h \frac{\mu}{1 + \mu r} \quad \text{when} \quad \mu \text{ is a non-zero number}. \tag{3.13}
\]

Here, \( \mu \) is the balancing proportion of the joint-production system as a virtual single production system; that is, a renormalization of \( m \):

\[
\mu = \frac{hZ^n p}{z_i Z_i^n l} \tag{3.14}
\]

The non-negative eigenvector \( h \geq 0 \) may then be regarded as the standard commodity equivalent to the modified joint-production system. We have therefore applied the same reasoning to the vertically integrated system to deduce the equivalent standard commodity. We have restricted the domain of the von Neumann system of \( [A, B] \) to a square system by adding fictitious commodities, and implemented a limiting primary factor \( l \) in the modified system.

### 3.1.4.7 Significance of the Standard Commodity

If we compare the results of the Sraffa theorem with the payoff function of the original von Neumann system, it holds from the Sraffa theorem for the eigenvectors \( s \) and \( p \) that:

\[
sAp = \frac{1}{1 + m(1 + r)}sp \quad \text{where} \quad \mu \text{ is a non-zero number} \tag{3.15}
\]
If we assume $B = I$ in the original von Neumann system, and apply the eigenvectors $s$ and $p$ to the payoff function of the von Neumann system, then we obtain:

$$x(I - \rho A)y = 0 \Rightarrow sAp = \frac{1}{\rho}sp$$

(3.16)

If we interpreted $m/(1 + m(1 + r))$ with $1/\rho$, expressions (3.15) and (3.16) could be regarded as equivalent. This means that the payoff function of the appropriately restricted von Neumann system by the condition $B = I$ could be represented in terms of Sraffa’s standard commodity by employing the balancing proportion $m$.

### 3.1.4.8 A Standard Commodity in an Extended System

The invertibility of $[B - A]$ was indispensable in setting up the Sraffa joint-production system. In fact, Sraffa (1960) skillfully implemented a new set of fictitious processes and commodities to complement the system to make the matrix $[B - A]$ square. This is a necessary condition for the invertibility of $[B - A]$. A fictitious process could realize a potential set of commodities. This realization may induce preparation of the new environment for either a newly emerged commodity or a new efficient process.

Suppose the actual production system is a general joint-production system. This does not necessarily confirm its solution in general. The non-existence of a solution (equilibrium prices) does not necessarily imply the impossibility of trading/production. Even without knowing the equilibrium prices, producers can trade with other producers under the double auction process underlying the changing environment.

### 3.1.4.9 A Standard Commodity to Measure the Profitability of the Fictitious Processes/Commodities in an Extended System

Each producer is assumed to have the right to post a new fictitious process or commodity. Here, we imagine the market as a field in which agents can post their rules and prices.

For example, if the number of commodities is greater than the number of processes in the existing production system, the system is under-determinant. If such a system successfully selected some of the posted fictitious processes to implement into itself, a standard commodity at that point in time could then be assured. In other words, a standard commodity can be fabricated in a proposed system. However, a chosen standard commodity may vary according to the method of selection of a set of new fictitious commodities. To generate a standard commodity in an extended system, there may exist a kind of combinatorial number corresponding to the
difference between the number of commodities $n$ and the number of processes $m$. Hence, the standard commodity of an extended system is not always unique.

A derived standard commodity in the above procedure will be a useful measure for producers who posted their fictitious processes to decide whether those processes are more profitable. That is to say, the payoff function of this virtual system will be evaluated by employing a derived standard commodity $s$ as its weight. In this sense, a standard commodity is regarded as the weight to measure the profitability of the fictitious processes and commodities in a virtually implemented system. The implication of the standard commodity given here must coincide with what Sraffa originally defined as the invariant measure of value.

The measure, even if hypothetical, could work as a reference point for a new system. This kind of measure could be regarded as the quasi-mean average of the concerned ensemble generated by the base production set. In this context, the mean average can be calculated by employing a standard commodity generated by the fictitious processes and commodities. This gives new insight into the Sraffa standard commodity in an extended system.\(^2\)

**Lemma 3.1.** *Updating a standard commodity* Competition among multiple standard commodities will orientate the future direction of the production system, which must provide feedback to its environment.

1. To guarantee the solution, some fictitious processor commodities should be added to the original system. However, the solution depends on the method of choosing each fictitious set. It may be multiple.
2. The profitability of the additional processes and commodities can be judged by the use of the standard commodity $s$ expected as a result of their addition.
3. Each standard commodity will give a virtual expected mean of the average profitability of the system.
4. The competing standard commodities will prepare for a new environment in which a new standard commodity can be updated.

We now turn to an intuitive scheme of this procedure. We can still use many detailed empirical analyses to compute the actual different standard commodities among national economies. These attempts will shed new light on the application of our new principle to empirical studies.

### 3.2 The Historical Background to Network Thinking in Economic Theory

Looking back through the literature, several scholars have discussed linear production systems similar to the von Neumann model, including Leontief (1966), Sraffa (1960), and Walras (1874). Sraffa’s contribution is perhaps especially important,

3.2 The Historical Background to Network Thinking in Economic Theory

because he analytically formulated the joint-productive recyclic system at almost the same time as von Neumann formulated his system. One criterion adopted by Sraffa was given by a set of macroeconomic distributive factors, including profit and wage rate.

It is easy for us to understand the inter-industrial input–output table as a network analysis. In the mid-twentieth century, many scholars worked on linear production systems, in view of the inequality approach. This is typically represented as the Duality Theorems of Systems of Linear Inequalities. These were believed to advocate an idea of equilibrium price in the context of the inter-industrial production system. However, we are now confronted by a constant process of innovation in the real world. Nevertheless, the idea of optimal prices guaranteed by the Duality Theorems merely freezes all the renovating processes of production, because the system can be fixed forever once the equilibrium price system has been attained. In other words, the idea of the optimal price system has rather shut off the dynamic properties of interactive activities. We should therefore examine again the approach to inter-industrial productive systems as part of a network analysis.

Georg Friedrich List (1789–1846) developed the idea of a national system of innovation. Arthur (2009) recommended national competitiveness in view of path dependency. By taking two instances, i.e., the power transmission line and hydroelectric power plants, Klaus Mainzer recently pointed out two industrialists who had successfully developed network thinking:

Oskar von Miller (1855–1934), a German engineer who founded the Deutsches Museum.

Walter Rathenau (1867–1922), the founder of Allgemeine Elektrizitäts-Gesellschaft (AEG), an electrical-engineering company, and foreign minister responsible for the Rapallo Treaty.

From this point of view, productive networks must be hierarchical, and their analysis will need complexity theory. This is explained further in Chap. 5. Here, we consider only the recycling of production.

3.2.1 The Recycling of Production

Figure 3.1 depicts a recycling system of production. In a real system of human production, each link of the recycling chain, i.e., each production process, must be currently associated with each labor input. If we remove an edge-node within the recycling operation in the figure, the system may degenerate into another one. This operation is called “truncation” and is the same as the procedure of rewiring nodes. However, we will need a more sophisticated rule than the assortative/dissortative one argued above. In Fig. 3.1, an addition of a new mode in the left diagram can give rise amore complicated relationship.

---

3 This opinion was given by him when I visited his office at TUM in Munich, March 2012.
Fig. 3.1 Productive networks
3.2 The Historical Background to Network Thinking in Economic Theory

3.2.2 The von Neumann Economy

We define the admissible set of activities $X$ in the von Neumann model of production as:

1. $x_t \in X$.
2. $x_tA = x_{t+1}B$.
3. $x_0B = X(0)$.

An activity mix is the mixed strategy that satisfies the above admissible set $X$. We denote the time series of feasible programs by $X_\theta$, and the time series of chosen programs by $C_\theta$:

Feasible programs: $X_\theta = \{x_{\theta t}(t = 0, 1, \ldots)\}$
Chosen programs: $C_\theta = \{c_{\theta t}(t = 0, 1, \ldots)\}$

The feasible programs are technologically bounded by the upper limit, so the maximal welfare level will be guaranteed.

3.2.3 Von Neumann’s Original Formulation

The economic system of production consists of dual systems of price and quantity. Production is achieved through a recycling network. In classic economics, we call this type of production “inter-industrial reproduction”. As a brief illustration, the production of outputs by recycling is taken to be achieved by a set of production processes by employing human labor and materials obtained from the outputs themselves. A productive process $i$ is therefore described as:

$$
\text{Production process } i : (a_i, l_i) \Rightarrow b_i
$$

(3.17)

$a_i$ is a set of materials $a_{ij}$ for process (or activity) $i$, $l_i$ is a labor input essential for process $i$, and $b_i$ is a set of outputs $b_{ij}$ contributed by process (or activity) $i$. $A = [a_{ij}]$ is an input matrix, $B = [b_{ij}]$ is an output matrix and $l = (l_i)$ is a labor vector. We can impose regularity conditions on our production system to guarantee solutions. We must then have a price system as the valuation measure of price, and a quantity system as the technically feasible activity measure:

price $p = (p_1, \ldots, p_n)$
quantity $q = (q_1, \ldots, q_m)$

Here $p_j$ represents the price of commodity $j$ and $q_i$ is an activity level $i$, which together construct evaluation systems that are the price-cost and supply-demand equation systems.
3.2.4 Classical Truncation Rules of Choice of Techniques

We define two types of feasibility: price and quantity.\(^4\) Price feasibility is defined as:

Price feasibility: Whether the price-cost equation of a subsystem is fulfilled with respect to non-negative prices at a given rate of profit \(r\): 
\[
[B - (1 + r)A]^L \leq l
\]
This means that sales minus input costs (including interest payments) is less than labor cost \(wl\).

Quantity feasibility is defined as:

Quantity feasibility: Whether the demand-supply equation of a subsystem is fulfilled with respect to non-negative activities at a given rate of growth \(g\):
\[
q[B - (1 + g)A] \geq d
\]
Here \(d_j\) represents a final demand of \(j\) consumed by labor. We call \(d = (d_j)\) a final demand or consumption vector. This means that supply minus intermediate input (with growth factor added) is greater than final demand.

Now truncation rules depend on:

Cost Minimization: Min \(ql\) subject to \([B - (1 + r)A]p \leq l\).
Income Maximization: Max \(dp\) subject to \(q[B - (1 + r)A] \geq d\).

3.2.5 Adaptive Plans in the von Neumann Economy

The society that chose \(c_{\theta,t}\) can uniquely specify the welfare level \(u(c)\). For convenience, this may be regarded as the expected net output or profit. In this case, we use the von Neumann payoff function as the welfare level. The welfare level that the program \(\theta\) accumulates during the period \((0, T)\) is then:
\[
U_\theta(T) = \sum_{t=0} u(c_{\theta,t}).
\]
Program \(\theta\) is preferred to any other program \(\theta'\) if:
\[
U_\theta(T) > U_{\theta'}(T)
\]
The optimal program is \(\theta\), satisfying the condition:
\[
glb_{c_{\theta} \in C} \lim_{T \to \infty} \sup [U_{\theta'}(T) - U_\theta(T)]
\]

\(^4\)See, for example, Thompson (1956), Nikaido (1968), and Morishima (1969) as for the basic ideas of the economic system.
An adaptive plan \( \tau \) is such that a production program \( C_\theta \) is chosen on the basis of information received from the economy \((E, \text{the environment})\). The problem is choosing a particular adaptive plan approaching an optimal program in a broader domain of the environment (Fig. 3.2).

### 3.2.6 The Market Mechanism as a Genetic Algorithm

We noticed earlier that the bid/ask mechanism for selecting an adaptive productive plan or program might be replaced by the bucket brigade algorithm as an internal mechanism. The process based on the evolutionary genetic algorithm moves to a new stage via a feedback loop from a consecutively modified environment. The environment in the genetic algorithm is interpreted as an environmental niche,
according to Holland (1992, p. 11; 1995, pp. 22–27). The field might be strategically chosen for the agents. This is called stage-setting in a genetic algorithm. Under such an implementation of the environmental niche, the environment for agents in this sense does not settle at a given landscape. Hence, the economic system will not necessarily be bound to an equilibrium state in which equilibrium prices dominate (Aruka 2012).

3.2.7 A System’s Eigenvector to Measure the Profitability of the Fictitious Processes/Commodities

Suppose the actual production system to be a general joint-production system. The system does not necessarily confirm its general solution.

3.2.8 Minimum Spanning Trees of the Industrial Network

In mathematical terms, a spanning tree of a connected and undirected graph is one that includes and connects all the graph’s vertices. The Minimum Spanning Tree (MST) is the smallest possible spanning tree for that graph. We can derive the MST from the cross-correlation matrices. Its diameter is the largest number of links that must be traversed to get from one node to another. This measurement gives an important index to monitor the transition between the different phases. The industry cluster may be regarded as more open if the degree of an MST is larger. By this definition, it follows that financial contagion spreads faster on a closer MST, and positive sentiment on a more open one.

3.2.8.1 Hub Industries in Japan

Cheong et al. (2012) discovered from the MST formations for each time period that the Chemicals and Electric Machinery industries are consistently the hubs through all five macroeconomic periods (Table 3.1). The authors also reached an interesting conclusion that “Going through all eight MSTs, we see that the growth hubs are fairly robust and persistent, and recover rapidly after short disappearances. The quality hubs, on the other hand, do not survive for very long. We take this to mean that Japanese investors are actually more optimistic than most people give them credit for”.


Table 3.1 Cross-correlations between the 36 Nikkei 500 Japanese industry indices over a 14-year period encompassing five macroeconomic periods

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Average</th>
<th>Maximum</th>
<th>MST diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire time series</td>
<td>0.216</td>
<td>0.507</td>
<td>0.805</td>
<td>5</td>
</tr>
<tr>
<td>Asian financial crisis (1997–1999)</td>
<td>0.234</td>
<td>0.485</td>
<td>0.767</td>
<td>5</td>
</tr>
<tr>
<td>Technology bubble crisis (2000–2002)</td>
<td>0.115</td>
<td>0.509</td>
<td>0.836</td>
<td>10</td>
</tr>
<tr>
<td>Economic growth (2003–2006)</td>
<td>0.126</td>
<td>0.498</td>
<td>0.819</td>
<td>9</td>
</tr>
<tr>
<td>Subprime crisis (2007–2008)</td>
<td>0.284</td>
<td>0.66</td>
<td>0.918</td>
<td>8</td>
</tr>
<tr>
<td>Lehman brothers crisis (2008–2010)</td>
<td>0.259</td>
<td>0.63</td>
<td>0.919</td>
<td>8</td>
</tr>
</tbody>
</table>


3.3 An Essential Characteristic of the Joint-Production System

An input–output relationship is a process \( i \) producing a single independent unit defined as a set of multiple inputs \( a_i = (a_{i1}, \ldots, a_{in}) \) associated with multiple outputs \( b_i = (b_{i1}, \ldots, b_{in}) \). Here \( a_{ij} \) has the input coefficient per capita output \( i \) (\( \sum_{j=1}^{n} a_{ij} \leq 1 \)) while \( b_{ij} \) is the output of each unit of process \( i \). For instance, a classical industry using some chemical reaction process was a typical joint-production system \( (b_i - a_i) \). When a certain link formation between interactive processes has been created, a spanned system of production among the links may not necessarily guarantee the coincidence of the number of processes (input) with the number of commodities (output) in view of a given technology. We have two ways to represent the production system: acyclic and cyclic.

3.3.1 An Acyclic Network of Production

The idea of an acyclic network is much broader than function. The function merely indicates a single outflow, even with multiple inflows. It will easily be seen that the neoclassical idea of production is too restrictive for a complex network of production. The diagrams shown below are all a broader case than the neoclassical production function (Figs. 3.3 and 3.4).

The alternative approach is a cyclic network of production. This idea is essential when we refer to a recyclic network. So far, we have considered the reproducibility of the production system. Here, the property of the input–output matrix \( b_i - a_i \) will play a decisive role in guaranteeing its reproducibility. Once the desired property of reproducibility—for example, the irreducible input matrix—has been established, the system will be replicated over time, by taking a certain price-cost system \( p = (p_1, \ldots, p_n) \) supporting the productive circulation of input–output exchanges:

\[
(b_i - (1 + r)a_i) p = wl_i \text{ for each process } i
\]  
(3.21)

\[
(B - (1 + r)A) p = wl \text{ in matrix form.}
\]  
(3.22)
Fig. 3.3 Multiple outputs from multiple inputs

Fig. 3.4 Multilayered process of production
### Table 3.2 Three-component table

<table>
<thead>
<tr>
<th></th>
<th>Primary</th>
<th>Secondary</th>
<th>Tertiary</th>
<th>Final demand</th>
<th>Export</th>
<th>Import</th>
<th>Domestic production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary industry</td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>-2</td>
<td>13</td>
</tr>
<tr>
<td>Secondary industry</td>
<td>3</td>
<td>175</td>
<td>58</td>
<td>156</td>
<td>56</td>
<td>-59</td>
<td>388</td>
</tr>
<tr>
<td>Tertiary industry</td>
<td>2</td>
<td>75</td>
<td>143</td>
<td>344</td>
<td>17</td>
<td>-11</td>
<td>570</td>
</tr>
<tr>
<td>Good value added</td>
<td>7</td>
<td>139</td>
<td>369</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic production</td>
<td>13</td>
<td>388</td>
<td>570</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note 1: Unit: trillion yen  
Note 2: The vertical direction represents the **cost composition of the merchandise**, and the horizontal direction the **sales destination of the merchandise**

Here $r$ is the interest rate, $w_i$ the wage rate, and $l_i$ the labor input. Given an effective demand $d = (d_1, \cdots, d_n)$, this manipulation is justified by finding:

$$\max dp$$  \hspace{1cm} \text{(3.23)}

Conversely, it will not hold if the particular price system does not happen. The idea of the optimal price system has therefore rather shut off the dynamic properties of interactive activities. Once the equilibrium price system has been established, the system is potentially fixed by the equilibrium *forever*.

Finally, we show the empirically estimated network by the model form of the input–output table (basic input–output table) for Fiscal 2005 (Three-Component Table) (Table 3.2).\(^5\)

The eigenvector centrality in the Three-Component Table, measured in 2005, is given as:

\[(0.333333, 0.333333, 0.333333)\] \hspace{1cm} \text{(3.24)}

The inter-industrial relationship appearing in the input–output table often exhibits the all-entering properties of the matrix in a strict sense (Fig. 3.5).\(^6\) The eigenvector centrality is inclined to be egalitarian. Instead of employing the eigenvector centrality, we can characterize the system by the closeness centrality of this three-character fundamental inter-industrial network:

\[(0.222222, 0.285714, 0.166667)\] \hspace{1cm} \text{(3.25)}

This is shown as a highlighted graph in Fig. 3.6.

---


\(^6\) This usually means that non-negative elements are found either in each row or each column. This situation is irrelevant to the idea of net producibility. *All-entering in a strict sense* means that most cells are non-positive. This property is always satisfied in the input–output matrix.
Fig. 3.5  The weighted adjacency graph of Table 3.2

Fig. 3.6  The closeness centrality of Table 3.2
3.3 An Essential Characteristic of the Joint-Production System

3.3.2 Criticism of the Traditional Approach

This formulation can be solved using the Duality Theorem of Linear Programming, but optimal solutions are often combined with a single industry operation. It is easily verified by simulation that the most profitable process is a simple one like oil production, not a more complex combination of multiple inputs. This may be why oil-producing countries often have high GDP per capita. This is also seen in international trade theory. Optimization will tend to make the most profitable process a simple one with just a few inputs. A more complex process cannot necessarily become more profitable. As long as we adopt the method of optimization in choosing technique, therefore, we will be unable to discuss how a complex system of production could be organized and evolve, so it may be wise to abandon the assumption of an artificial optimal/equilibrium price system.

We have, however, a useful insight from the traditional approach. Specifically, following Sraffa (1960), we can focus on the use of eigenvectors in the linear production system. In the 1930s, Sraffa was ready to identify the maximal (or principal) non-negative eigenvector with the invariant measure of the production of commodities by means of commodities. Sraffa happened to find virtually the same content as the Perron-Frobenius Theorem on the nonnegative eigenvectors on a real square matrix.\(^7\) In his own economic modeling. Given a linear production system, we can find it uniquely, by fulfilling some conditions for a net producible system. We will then encounter the same property as the largest eigenvector, where the composition of industries on the input side is proportionate to that on the output side. If \(w\) is set as \(0\), we get:

\[
Ap = 1/(1 + r)Bp, \text{ i.e.,} \quad Ap = \lambda Bp
\]  

This is the eigenvector equation if \(B\) is a unit matrix \(I\). In the following, for simplicity, we assume \(B = I\).

3.3.2.1 A Numerical Example

For convenience, we use a numerical example. We specify the numerical values in an input matrix \(A\):

\[
A = \begin{pmatrix}
0.1 & 0.3 & 0 \\
0.7 & 0.5 & 0 \\
0 & 0.1 & 0.5 \\
\end{pmatrix}
\]  

\(^7\)See Perron (1907) and Frobenius (1912). See, for example, Nikaido (1968) for a fine proof of the nonnegative solutions in the economic system.
This input system can be net producible, i.e., produce a net surplus. This matrix has the eigenvectors and eigenvalues:

Eigenvectors:

\[ v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0.376638 & 0.878823 & 0.292941 \\ 0 & 0 & 1 \\ 0.703526 & -0.703526 & 0.100504 \end{pmatrix} \] (3.29)

associated with eigenvalues:

\[ (u = \{u_1, u_2, u_3\}) = \{0.8, 0.5, -0.2\} \] (3.30)

The largest positive eigenvector is judged to be the first mode \( v_1 \), as it has the largest eigenvalue \( u_1 \) (scalar). In this view of eigenvectors, the second sector in the first mode will give the dominant incidence of the system. According to Sraffa’s insight, this system’s valuation will not be affected by any disturbance of distributional factors, as long as the system is reduced to the eigen-system maintaining the proportionality between the input and output components. In other words, the largest eigenvector will behave like a fixed point as distributional variables \( r \) and \( w \) fluctuate. In our example, we measure the system by the composite commodity defined by the first mode \( v_1 \), but not by a simple component like wheat. In this case, therefore, the second sector must be influential among the sectors used for valuation. However, this idea will also remove any opportunities for the production system to evolve.

The input coefficients represent demand links; that is to say, \( a_{ij} \) means sector \( i \) requires \( j \) for self-activation. We may then regard our input matrix \( A \) as a weighted adjacency matrix, and it may represent the next productive (inter-industrial) network.

Each vertex is regarded as a production sector or process of production. This network is easily verified using a cyclic graph.

We can therefore apply the cyclic network to the eigenvector centrality. By taking the transpose of matrix \( A \), i.e., an adjacency matrix, the eigenvector centrality \( x \) may be calculated:

\[ x = \frac{1}{\lambda} A'x \] (3.31)

Here \( A' \) is the transpose of matrix \( A \). In economic terms, \( A' \) represents supply links, while \( A \) represents demand links. \( x \) represents an eigenvector in terms of the activity level of the sector. We have already seen that the largest eigenvector of the input matrix is:

\[ (0.376638, 0.878823, 0.292941) \] (3.32)
### 3.3 An Essential Characteristic of the Joint-Production System

#### Fig. 3.7 The cyclic network of the numerical example

#### Fig. 3.8 Eigenvector centrality

#### Fig. 3.9 Degree centrality

The largest eigenvector of the adjacency matrix turns out to be:

\[(0.5, 0.5, 0)\]  \hspace{1cm} (3.33)

We have previously called the discrepancy between the input matrix and its transposed matrix the dual instability between the price system \(p\) and the quantity system \(x\). Now we have again encountered the same kind of problems in terms of the network analysis. If we look only at valuation, the second sector incidence in our numerical example will be the largest. However, if we look at eigenvector centrality, the first and second sectors will be equally attractive. In other words, the network incidence in terms of the weighted adjacency matrix indicates that the first and second modes have equal incidences. The modes do not correspond exactly to the actual sectors, but each mode is constituted by the actual sector components (Figs. 3.7, 3.8, and 3.9).

We then discover that degree centrality is located on:

\[(2, 3, 1)\]  \hspace{1cm} (3.34)

We therefore obtain a similar result for the demand links (the input matrix), in terms of ordering between the sectors.
We pointed out above that using the largest eigenvalue as the invariant measure might prevent any opportunities for the production system to evolve. However, the stability of the demand links differs from that of the supply links. This divergence will generate some dilemmas, because the given supply link will not necessarily support new demand links, if any should be created to a particular node. Node creating will affect the demand and supply links differently. We focus on the way the network will evolve by using some numerical examples.

3.4 An Empirical Study Using Input–Output Tables

The inter-industrial input–output table is readily available to provide empirical information. This table has common classification rules, so the number of industries is usually fixed for particular periods, though some revisions of classifications are intermittently incorporated, and groupings may therefore change periodically.

The Input–Output Tables in Japan, released every 5 years since 1955, are the most important statistics for the revision of standards and extended estimation of GDP for the calculation of the National Accounts by the Japanese Cabinet Office. The tables are produced by several Government agencies. The table is based on Basic Sector Tables, with 520 rows × 407 columns. The tables are of various sizes ranging from 13 to 34 and 108 to 190-sector classifications. We used 13 or 34-sector tables for convenience, covering the years 1995, 2000, and 2005. There was no 34-sector table in 2000, as it only has 32 sectors, because the items “14 Information and communication electronics equipment” and “15 Electronic components” were not explicitly given as formal sectors. We can regard these items as emerging links between 2000 and 2005. In other words, the input–output system structurally evolved in this time. The items of sector classification are shown in Table 3.3.

3.4.1 The First Step Towards Empirical Input–Output Analysis

We have argued an elementary analysis in a general case of a joint-productive system, which permits the system to produce a set of multiple outputs in a single process of production. However, the empirical data such as the input–output table

---

8The current members of the program are the Ministry of Internal Affairs and Communications (coordinator), the Cabinet Office, the Financial Services Agency, the Ministry of Finance, the Ministry of Education, Culture, Sports, Science and Technology, the Ministry of Health, Labor and Welfare, the Ministry of Agriculture, Forestry and Fisheries, the Ministry of Economy, Trade and Industry, the Ministry of Land, Infrastructure, Transport and Tourism, and the Ministry of Environment. See http://www.stat.go.jp/english/data/io/outline.htm.
### Table 3.3 Sector classifications

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>01 Agriculture, forestry and fishery</td>
<td>01 Agriculture, forestry and fishery</td>
<td>01 Agriculture, forestry and fishery</td>
</tr>
<tr>
<td>02 Mining</td>
<td>02 Mining</td>
<td>02 Mining</td>
</tr>
<tr>
<td>03 Manufacturing</td>
<td>03 Beverages and foods</td>
<td>03 Beverages and foods</td>
</tr>
<tr>
<td>04 Construction</td>
<td>04 Textile products</td>
<td>04 Textile products</td>
</tr>
<tr>
<td>05 Electric power, gas and water supply</td>
<td>05 Pulp, paper and wooden products</td>
<td>05 Pulp, paper and wooden products</td>
</tr>
<tr>
<td>06 Commerce</td>
<td>06 Chemical products</td>
<td>06 Chemical products</td>
</tr>
<tr>
<td>07 Finance and insurance</td>
<td>07 Petroleum and coal products</td>
<td>07 Petroleum and coal products</td>
</tr>
<tr>
<td>08 Real estate</td>
<td>08 Ceramic, stone and clay products</td>
<td>08 Ceramic, stone and clay products</td>
</tr>
<tr>
<td>09 Transport</td>
<td>09 Iron and steel</td>
<td>09 Iron and steel</td>
</tr>
<tr>
<td>10 Communication and broadcasting</td>
<td>10 Non-ferrous metals</td>
<td>10 Non-ferrous metals</td>
</tr>
<tr>
<td>11 Public administration</td>
<td>11 Metal products</td>
<td>11 Metal products</td>
</tr>
<tr>
<td>12 Services</td>
<td>12 General machinery</td>
<td>12 General machinery</td>
</tr>
<tr>
<td>13 Other activities</td>
<td>13 Electrical machinery</td>
<td>13 Electrical machinery</td>
</tr>
<tr>
<td>N/A</td>
<td>14 Information and communication electronics equipment</td>
<td>14 Information and communication electronics equipment</td>
</tr>
<tr>
<td>N/A</td>
<td>15 Electronic components</td>
<td>15 Electronic components</td>
</tr>
<tr>
<td>14 Transportation equipment</td>
<td>16 Transportation equipment</td>
<td>16 Transportation equipment</td>
</tr>
</tbody>
</table>

(continued)
3. Network Analysis of Production and Its Renewal

Table 3.3 (continued)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15 Precision instruments</td>
<td>17 Precision instruments</td>
<td></td>
</tr>
<tr>
<td>16 Miscellaneous manufacturing products</td>
<td>18 Miscellaneous manufacturing products</td>
<td></td>
</tr>
<tr>
<td>17 Construction</td>
<td>19 Construction</td>
<td></td>
</tr>
<tr>
<td>18 Electricity, gas and heat supply</td>
<td>20 Electricity, gas and heat supply</td>
<td></td>
</tr>
<tr>
<td>19 Water supply and waste disposal business</td>
<td>21 Water supply and waste disposal business</td>
<td></td>
</tr>
<tr>
<td>20 Commerce</td>
<td>22 Commerce</td>
<td></td>
</tr>
<tr>
<td>21 Finance and insurance</td>
<td>23 Finance and insurance</td>
<td></td>
</tr>
<tr>
<td>22 Real estate</td>
<td>24 Real estate</td>
<td></td>
</tr>
<tr>
<td>23 Transport</td>
<td>25 Transport</td>
<td></td>
</tr>
<tr>
<td>24 Information and communications</td>
<td>26 Information and communications</td>
<td></td>
</tr>
<tr>
<td>25 Public administration</td>
<td>27 Public administration</td>
<td></td>
</tr>
<tr>
<td>26 Education and research</td>
<td>28 Education and research</td>
<td></td>
</tr>
<tr>
<td>27 Medical service, health, social security and nursing care</td>
<td>29 Medical service, health, social security and nursing care</td>
<td></td>
</tr>
<tr>
<td>28 Other public services</td>
<td>30 Other public services</td>
<td></td>
</tr>
<tr>
<td>29 Business services</td>
<td>31 Business services</td>
<td></td>
</tr>
<tr>
<td>30 Personal services</td>
<td>32 Personal services</td>
<td></td>
</tr>
<tr>
<td>31 Office supplies</td>
<td>33 Office supplies</td>
<td></td>
</tr>
<tr>
<td>32 Other activities</td>
<td>34 Other activities</td>
<td></td>
</tr>
</tbody>
</table>


using industrial sectors require the special but practical idea of a single output from a single process of production. This peculiarity of the input–output analysis therefore limits our tool set for inter-industrial network analysis.

3.4.1.1 The Closeness Centrality

In Sect. 3.2, we mentioned that there is often an egalitarian eigenvector centrality in the input–output table. We therefore employ the idea of closeness centrality to characterize the inter-industrial network. The definition of closeness centrality (see
Freeman 1978/1979; Opsahl et al. 2010; Wasserman and Faust 1994) is:

\[
\text{closedness}(i) = \sum_j [d_{ij}]^{-1} = \frac{1}{\sum_j d_{ij}} \tag{3.35}
\]

Here \(i\) is the focal node and \(j\) is another node in the targeted network, while \(d_{ij}\) is the shortest distance between these two nodes.\(^9\)

### 3.4.1.2 The Historical Transitions in the 13-Sector Classification

Although the definitions have not remained entirely constant over time, the 13-sector classification is consistent enough over the last three periods, 1995, 2000, and 2005. We look first at the transition between the three periods. There is a fixed set of graph communities like categories \(\{1, 3, 4\}\) and \(\{2, 5, 9\}\) across all three periods. However, components \(\{6, 7, 8, 10, 11, 12, 13\}\) have separated into subsets. We can show the evolution of the weighted adjacency graph highlighted by the closeness centrality across the period as Figs. 3.10, 3.11, and 3.12.

We can also summarize the transitions among the graph communities in table form, in Table 3.4.

### 3.4.1.3 An Application to a More Expanded Case: 32 to 34-Sector Classification

We can apply the same analysis of closeness centrality and graph communities to the 32-sector case in 2000 (Fig. 3.13) and the 34-sector case in 2005, producing diagrams of the highlighted graph of the weighted adjacent matrix, and the transitions among the graph communities. See Fig. 3.14 and Table 3.5.

The operation to add two nodes, 14 and 15, does not imply real change for the whole system of production, because the actual production network has not changed at all after this statistical procedure. The added nodes require a nominal change of the existing wiring network, with almost immediate replacement of link attachments. We may therefore interpret these added nodes alongside the missing nodes in the existing inter-industrial network. By renormalization, these two missing nodes have been revealed. If the added nodes are missing and to be renewed, i.e., disconnected, the system under observation could be a closed system, and we may regard a newly emerging community as a newly wired set of links, irrespective of the real change in the whole value.\(^10\)

---

\(^9\)In this definition, we will be faced with the difficulty of infinity if the system under consideration is disconnected, but another definition is possible to avoid this. See Opsahl et al. (2010).

\(^10\)Our observation, either in terms of technological property or of data availability, may be supplemented by Ohkubo’s rule. Ohkubo and Yasuda (2005) and Ohkubo et al. (2006) proved the
Fig. 3.10  The highlighted graph of the weighted adjacent matrix in 1995

Fig. 3.11  The highlighted graph of the weighted adjacent matrix in 2000
Fig. 3.12 The highlighted graph of the weighted adjacent matrix in 2005

<table>
<thead>
<tr>
<th>Year</th>
<th>Graph communities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>{{6, 7, 8, 10, 11, 12, 13}, {1, 3, 4}, {2, 5, 9}}</td>
</tr>
<tr>
<td>2000</td>
<td>{{7, 8, 11, 13}, {1, 3, 4}, {2, 5, 9}, {6, 10, 12}}</td>
</tr>
<tr>
<td>2005</td>
<td>{{6, 8, 10, 12}, {1, 3, 4}, {2, 5, 9}, {7, 11, 13}}</td>
</tr>
</tbody>
</table>

It is immediately seen that the community {2, 22, 23, 24, 25, 26, 28, 30, 31, 32, 34}, by renaming previous items, has reduced itself to a smaller sub-community :{21, 23, 24, 26, 27, 30, 31, 34}. On the other hand, an emerging community {13, 14, 15, 17, 28} around the two nodes 14 and 15 has absorbed 13, 17, and 28, which previously belonged elsewhere. We have successfully traced a new community formation in the inter-industrial network.

Finally, in Figs. 3.15 and 3.16, we show a set of graphical presentations corresponding to Table 3.3.

equivalence between a Polya urn process and a renewal of the network spanned by a closed number of links. Taking into account that a finite set of productive processes on the input–output table is given, and using Ohkubo’s rule, we can show a renewal process of the network of production under the preferential attachment within the closed set of links. Here we regard a replacement of link attachment as a replacement of a productive process.
Fig. 3.13  The highlighted graph of the weighted adjacent matrix in 2000

Fig. 3.14  The highlighted graph of the weighted adjacent matrix in 2005
### Table 3.5 Transitions among the graph communities in the 32/34-sector case

<table>
<thead>
<tr>
<th>Year</th>
<th>Graph communities</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>{2, 20, 21, 22, 23, 24, 26, 28, 29, 30, 32}, {3, 4, 5, 6, 16, 19, 27, 31},</td>
</tr>
<tr>
<td></td>
<td>{8, 9, 10, 11, 17}, {12, 13, 14, 15, 25}, {1, 7, 18}</td>
</tr>
<tr>
<td>2000*</td>
<td>{2, 22, 23, 24, 25, 26, 28, 30, 31, 32, 34}, {3, 4, 5, 6, 18, 21, 29, 33},</td>
</tr>
<tr>
<td></td>
<td>{8, 9, 10, 11, 19}, {12, 13, 16, 17, 27}, {1, 7, 20}</td>
</tr>
<tr>
<td>2005</td>
<td>{21, 23, 24, 26, 27, 30, 31, 34}, {2, 7, 8, 20, 25}, {13, 14, 15, 17, 28},</td>
</tr>
<tr>
<td></td>
<td>{5, 18, 22, 33}, {9, 11, 12, 19}, {1, 3, 32}, {4, 6, 29}, {10}, {16}</td>
</tr>
</tbody>
</table>

*Re-numbers to reflect classification in 2005

### 3.4.2 A Further Consideration for Empirical Studies of the Inter-Industrial Network

In the input–output table, new links in the input–output analysis will occur when a new classification rule is employed, as we have seen by comparing the 32-sector classification in 2000 and the 34-sector classification 5 years later. As long as we are focused on the input–output analysis, therefore, it is difficult for us to analyze a true process of newly created links, i.e., a truly innovative process. In our empirical framework, we are forced to use an indirect approach to detect innovation and its effect on the network using the inter-industrial input–output tables.

We now mention two considerations for future work.

#### 3.4.2.1 External Links from Export Industries

In this chapter, we have not explicitly introduced the price system, so we have not considered the causes of technological changes driven by the price system. We can temporarily examine some of the effects from the demand side through export industries. The inter-industrial network analysis compactly summarizes the **production inducement coefficient** due to export from each industry, and the effects caused by the export activities. Roughly speaking, the average coefficient of this during the last decade is estimated as 2.\(^{11}\) Each unit of export can contribute about twice its export size to its activity level. The change in exports over different industries will induce a new wiring set of links independently, irrespective of domestic demand links. The export industry is therefore an important demand link that contributes directly to the domestic inter-industrial network. The next step is to introduce the components of export directly matched to the domestic supply network.

Fig. 3.15  The set of re-combinatorial wiring communities of the inter-industrial network in 2000

Fig. 3.16  The set of re-combinatorial wiring communities of the inter-industrial network in 2005
Fig. 3.17  The component distributions of eigen-sectors in the 13-sector table in 2005
3.4.2.2 The Trade-Off Relationships Between Industrial Components or Communities

Recently, it has been discernible that financial activities may invalidate the activities of other real industrial sectors. By applying covariance analysis to the input coefficient matrix of the input–output table, we can characterize each eigen-mode by its principal component with the largest value. If the principal component is the seventh element, the eigen-mode may be dominated by the finance industry; that is, the seventh in Table 3.3. We can then find opposite signs in each eigen-mode. In such a case, the dominant component may react in the opposite direction to other associated industries with opposing signs. These observations must be helpful to provide a rough profile of the swap or trade-off between different industrial activities.

Figure 3.17 shows the component distributions in each eigen-sector. This type of analysis will also contribute to finding a potential linkage to change the development of the inter-industrial network.

The network analysis has become popular in this century. The analysis of hierarchically nested structures will be developing. As the network analysis advances, the traditional subjects of economics will be recovered by employing a newly emerging network analysis. In fact, some of classical economists virtually shared with the same focus as the modern network analysis.

References


Covariance analysis of household demand in Japan assisted by the random matrix theory can be found in Aruka et al. (2013).
Chapter 4
Matching Mechanism Differences Between Classical and Financial Markets

Abstract The world currently faces a global financial crisis following massive breakdown of the financial sector, at least in part because of deregulation. But what does this mean for economics? We explained in Chap. 1 that the modern financial market differs in many ways from the classical economic idea of a market. A modern financial exchange is a system of heterogeneous interactions, all with different strategies. The participants may no longer be regarded as a homogeneous agent, subject only to the common rationality principle. Traders’ strategies are confined by regulations setting out the complicated rules and customs for auctions. A simultaneous move of ask and bid may be allowed. A strategy employing the market order without specifying the limit order may also be allowed. The market could accept any type of order, whether intelligent or non-intelligent. Non-intelligent agents may even be winners. Behavioral considerations, based on game theory, may be unhelpful or even useless in the market as it truly exists. Actual transaction customs are based not only on institutions but also computer servers. We therefore also need to examine the design of AI-based servers as well as transaction algorithms. This may lead us to re-examine the features of the free market, and in particular the financial one. Over recent years, we have been able to successfully examine a set of features of the market system by developing an AI simulator for the futures stock market, which is called U-Mart. In the light of this work, we will discuss an essential structure for the coordination of supply and demand in the free financial market system.
4.1 Reconsidering the Law of Supply and Demand in the Free Market

Michio Morishima was a great twentieth-century economist, rare in recognizing the function of auctions (Morishima 1984). Many economists describe an auction mechanism as either a batch or Itayose auction, since these are both realistic forms. In a pure form, however, the idealized auction mechanism is different. We therefore first consider the auction mechanism proposed by Morishima. He suggested that an actual auction may be incomplete. In this “ideal Walrasian auction”, no trades will be contracted until equilibrium has been confirmed. The auctioneer must behave neutrally and provide no information to any one trader about the situation of others.

Maurice Potron (1872–1942) was a French Catholic priest who was also interested in mathematical economics, thanks to his interest in the stuff of ordinary life, such as bread (Bidard et al. 2009). Potron used the Perron-Frobenius theorem (Frobenius 1908, 1909; Perron 1907) in his model as early as 1911, and in doing so anticipated some aspects of Frobenius’s (1912) generalizations. The importance of this theorem for economic theory was really only acknowledged after the Second World War. Since they were writing in the late nineteenth and early twentieth centuries, it is perhaps not surprising that these economists focused on bread as the main object of an auction. In this chapter, we will show the transition of trading methods in parallel with the change in what is being traded, from real to financial commodities.

4.1.1 The Classical Auction with Complete Ignorance of Others’ Preferences

The market is normally regarded as unfair if any participating agent knows in advance the situation or preferences of any other. A classical auctioneer coordinates a set of orders, but without collecting all asks/bids from the participating agents. In this context even a sealed bid may violate the fairness of the market. The classical theory of auctions therefore works on the assumption of complete ignorance. This implies that the auctioneer does not employ any cumulated form of supply or demand, i.e., the supply/demand curve. The visible curves may no longer be compatible with the assumption of ignorance. We now examine a numerical example (Morishima 1984).

Without ignorance of personal information, we can immediately construct the supply curve by combining the orders at each price starting with the lowest (the market order), and the demand curve by combining the orders starting with the highest price (the market order). These curves are therefore independently derived. We call this the collective method. Using a numerical example we then verify that the classical procedure results in the same equilibrium as the collective method (Table 4.1).
4.1 Reconsidering the Law of Supply and Demand in the Free Market

### Table 4.1 Classical auction 1

<table>
<thead>
<tr>
<th>Seller</th>
<th>Price</th>
<th>Quantity</th>
<th>Buyer</th>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>MO</td>
<td>300</td>
<td>E</td>
<td>MO</td>
<td>400</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>300</td>
<td>F</td>
<td>23</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>21</td>
<td>200</td>
<td>G</td>
<td>22</td>
<td>300</td>
</tr>
<tr>
<td>D</td>
<td>22</td>
<td>100</td>
<td>H</td>
<td>20</td>
<td>300</td>
</tr>
</tbody>
</table>

MO: Market Order

*Source:* Table 8 in Aruka and Koyama (2011)

<table>
<thead>
<tr>
<th>Asking price</th>
<th>Ask</th>
<th>Bid</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>~19</td>
<td>300</td>
<td>400</td>
<td>+100</td>
</tr>
<tr>
<td>20</td>
<td>B300</td>
<td>H300+MO100</td>
<td>+H100</td>
</tr>
<tr>
<td>21</td>
<td>C200+H200</td>
<td>F100+G300</td>
<td>±0</td>
</tr>
</tbody>
</table>

Note: MO will be matched first, then H. H200 purchased at ¥20 is resold at ¥21, because H only desired to buy at the price lower than ¥21.

#### 4.1.1.1 A Numerical Simulation

Provided that the auctioneer holds to the ignorance of personal information, we do not have any means of combining prices in this way. We therefore need a different method to find an equilibrium.

Market custom prescribes the initial settlement of market orders, leaving an excess demand of 100. Custom also requires the asking price to be quoted from the lowest, and bids from the highest. This custom regulates matching to control direction. It is readily apparent that custom could guide sellers and buyers to match at an equilibrium price. The demand may then be appropriated from Seller B, who wants to sell 300 at ¥20. This creates availability of 200 at ¥20. At ¥20, Buyer H can purchase 200 but still wants a further 100. As the quoted price is raised to ¥21, H may prefer not to purchase. H can, however, cancel the obtained right to 200 at ¥21 because H demanded only 200 at ¥20 or less. Buyer H must then become Seller H of 200 units as the quoted price becomes ¥21. At a price of ¥21, there are $C + H = 400$ units on the sellers’ side and $F + G = 400$ units on the buyers’ side. The price of ¥21 is therefore an equilibrium one, and cannot be blocked by any further ask or bid. If D tries to sell 100 at ¥22, nobody will buy it. If B tries to sell 300 units at ¥20, there will be no other seller at ¥20, so B cannot fulfill the total demand for 400 units. ¥21 is therefore an equilibrium price. This is shown in Fig. 4.1.

#### 4.1.1.2 An Equilibrium Quantity

Until arriving at ¥22, the total orders on the sellers’ side may vary by price from A100 at ¥19; to $100 + B300 + H200 = 600$ at ¥20 and $600 + C200 = 800$ at ¥21. The total orders from buyers may vary from $E400 + F100 + G300 + H300 = 1,100$ at
the price ¥19 to 1100 — H200 = 900 at ¥20, 900 — H100 = 800 at ¥21 800 at ¥22; and 800 — G300 = 500 at ¥23. At ¥21, A, B and C have therefore sold their stocks while E, F and G have purchased. The right-hand side of Fig. 4.2 shows a matching process that may lead to the same equilibrium as the supply and demand curves based on the collective method, except for multiple equilibria. We employ the rule of re-contracts and evaluate which orders are finally feasible at the equilibrium price, after eliminating the previous re-sales or re-purchases. In the collective method, we do not need to do this.

Agent H, who can become a seller instead of a buyer by reselling his purchases, has a particular role.
4.1 Reconsidering the Law of Supply and Demand in the Free Market

Under the assumption of ignorance, as we saw above, Agent H must recover his income by reselling previously purchased stock at the previous price. This kind of transaction is irrelevant to other activity. This handling would be invalid if the ignorance clause was relaxed. In other words, this additional behavior of reselling will be deleted in the collective method by the independent supply/demand curve. Other steps are required to detect equilibrium with the assumption of ignorance.

4.1.2 Auctions in the Financial Market

As we saw, the above tâtonnement may derive the same equilibrium as the collective method, i.e., the supply and demand curves, as long as the local equilibrium is the same. The collective method provides the participants with all the information about supply and demand. This knowledge is sufficient to activate agent reactions to the configuration of orders to bring about the second stage of coordination. This may no longer be based on the original preferences but instead interactive effects among the agents.

4.1.2.1 A Welfare Implication in Financial Market Auctions

We can suggest an alternative rule, the financial market rule. Here, a buyer could become a seller, or vice versa. In the financial market, “buyer” or “seller” are temporal forms, which change over time. The transition from buyer to seller must be realized in the financial market. In the example above, the buyer who temporarily gives up his demand can wait for a future price rise to sell, so this is only a bifurcation step that divides the welfare meanings of trades between the Walrasian and financial steps. In the latter, an agent wants extra profit, so must bet or gamble on rising prices.

In the Walrasian tâtonnement, however, any agent only expects to fulfill his original desire, following the classical economic idea of fairness. If we move away from this, we can easily diverge the market allocation from the original set of demand and supply. This divergence may often induce the market to float away from the real distribution of supply and demand, because many trades realized may be the by-products of psychological interactions among agents. Even in these interactions, the collective method may be constructed, as in a batch or Itayose auction. The collectively composed supply and demand curves may not have any detached ground. The agent’s expectation drives strategy (buy, sell, or nothing) as well as the reactions of other agents. Added orders are the results of interactive
summation, so the collective curves may no longer reflect the classical curves constructed using the agents’ independent preferences (Fig. 4.3).

According to Arthur (2009, p. 154), it is not quite correct to say that derivatives trading ‘adopted’ computation:

That would trivialize what happened. Serious derivatives trading was made possible by computation. At all stages from the collection and storage of rapidly changing financial data, to the algorithmic calculation of derivatives prices, to the accounting for trades and contracts computation was necessary. So more accurately we can say that elements of derivatives trading encountered elements of computation and engendered new procedures, and these comprised digitized derivatives trading.

Large-scale trading of derivative products took off as a result.

4.1.2.2 The Historical Alteration of Trading Methods from the Classical Collective Method to the Double Auction

As mentioned at the beginning of Chap. 3, the keen interest in the theory of price and production that dominated the 1920s and 1930s had decreased drastically by the early 1970s. Similarly, there had also been an alteration in trading methods, from the classical collective method to the double auction. We use the phrase ignoratio elenchi for this change, which is often characterized as “irrelevant conclusion”, indicating a logical fallacy that consists in apparently refuting an opponent, while actually disproving something not actually asserted for either market:

The collective method could be valid in some cases for both the Walrasian free market and the contemporary financial market to find equilibrium. We must, however, be careful as the results obtained have different meanings for
4.1 Reconsidering the Law of Supply and Demand in the Free Market

Exchange System

Forms
Futures
Spot

Auction Rules
Double
Batch

Ask
Brokers

Bid

Buy

Sell

Consumers

Producers

Fig. 4.4 Exchange system

the two markets. The financial market is the institution or the set of rules required to activate the trading, and may never guarantee certain welfare for the participating agents. In futures trading an element of gambling is introduced. If we regard both free markets as sharing the same functions, we will have ignoratio elenchi.

If any agent has not decided whether to act as seller or buyer before entering the market, that agent’s preference could simply depend on interactive reactions. If the agent also has dealings in the futures trade with leverage, they can only play a psychological game in seeking profit. This game is then irrelevant to the real economy, even though it contains some indirect linkage to reality. These collective dynamics are irrelevant to optimal allocation, but an essential feature of the futures market. As econophysics has proved, therefore, high volatility produces still higher volatility.

The collective method for determining supply and demand can no longer hold in the general ask/bid auction or double auction. This means that we will struggle to obtain any useful analysis of the financial market using the supply and demand curves.

In short, as we noted before, price no longer plays a crucial role in the economy, and the equilibrium price system is no longer the sole decisive factor in the fulfillment of trades. This suggests that a replacement of the trading method will occur, which has been seen in the move away from batch auctions towards continuous double auctions. This is also reflected in the metamorphosis of production shown in a service-dominant economy (Fig. 4.4).

4.1.2.3 The Ask and Bid Auction

The initial price on resumption after a trade halt—for example, overnight on stock exchanges—is established by the ask/bid method. This is usually employed on
Table 4.2  Ask and bid auctions

<table>
<thead>
<tr>
<th>MO for sell</th>
<th>Ask (Sell)</th>
<th>Price</th>
<th>Bid (Buy)</th>
<th>MO for buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1,798~</td>
<td>30</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>82</td>
<td>1,798</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>73</td>
<td>1,793</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>38</td>
<td>1,779</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>44</td>
<td>1,778</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>91</td>
<td>1,776</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1,770</td>
<td>37</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>93</td>
<td>1,763</td>
<td>6</td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>57</td>
<td>1,759</td>
<td>23</td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>1,758</td>
<td>34</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1,741</td>
<td>71</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1,733</td>
<td></td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1,729</td>
<td></td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1,719</td>
<td></td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1,684</td>
<td>21</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>~1,684</td>
<td></td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3  Supply and demand in ask/bid auctions

<table>
<thead>
<tr>
<th>Supply</th>
<th>Price</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>493</td>
<td>1,798~</td>
<td>30</td>
</tr>
<tr>
<td>493</td>
<td>1,798</td>
<td>30</td>
</tr>
<tr>
<td>411</td>
<td>1,793</td>
<td>30</td>
</tr>
<tr>
<td>338</td>
<td>1,779</td>
<td>30</td>
</tr>
<tr>
<td>300</td>
<td>1,778</td>
<td>30</td>
</tr>
<tr>
<td>256</td>
<td>1,776</td>
<td>30</td>
</tr>
<tr>
<td>165</td>
<td>1,770</td>
<td>67</td>
</tr>
<tr>
<td>165</td>
<td>1,763</td>
<td>73</td>
</tr>
<tr>
<td>72</td>
<td>1,759</td>
<td>96</td>
</tr>
<tr>
<td>15</td>
<td>1,758</td>
<td>130</td>
</tr>
<tr>
<td>15</td>
<td>1,741</td>
<td>201</td>
</tr>
<tr>
<td>15</td>
<td>1,733</td>
<td>201</td>
</tr>
<tr>
<td>15</td>
<td>1,729</td>
<td>201</td>
</tr>
<tr>
<td>15</td>
<td>1,719</td>
<td>201</td>
</tr>
<tr>
<td>15</td>
<td>1,684</td>
<td>222</td>
</tr>
<tr>
<td>15</td>
<td>~1,684</td>
<td>222</td>
</tr>
</tbody>
</table>

stock exchanges for the opening and closing prices in both morning and afternoon sessions. The price when a special quote is indicated is also determined by this method. The rest of the time, the Tokyo Stock Exchange at least uses the double auction method (Tables 4.2 and 4.3; Fig. 4.5).
4.1.2.4 English Auctions and Dutch Auctions

There are two main alternative auction rules: Dutch and English. Given a single order book, two kinds of equilibrium can usually be achieved. This result suggests that the idea of equilibrium depends on the rule employed. In Dutch auctions, the auctioneer quotes the prices in descending order. In English auctions, they are quoted in ascending order. Let \( s_i(p) \) be the quantity supplied by agent \( i \) at price \( p \), and \( d_j(p) \) the demand of agent \( j \) at price \( p \). The necessary conditions for establishing equilibrium are:

**English auction** The total balance of supply at the quoted price is smaller than the total balance of demand at the same quoted price.

\[
\sum_i s_i(p) \geq \sum_j d_j(p)
\]  

(4.1)

**Dutch auction** The total balance of supply at the quoted price is greater than the total balance of demand at the same quoted price.

\[
\sum_i s_i(p) \leq \sum_j d_j(p)
\]  

(4.2)

It is clear that equilibrium is always unique for the strict equality case. In these cases, we need **piecemeal stipulation**. In an auction session, any orders remaining after settlement will be carried over to the next session. The equality case will not necessarily hold in the discrete price change actually employed. We will only guarantee a unique equilibrium in the case of imaginary continuous change.

Now we take an example to show two equilibria. In this numerical example, for simplicity, we drop the market orders on both sides of ask and bid (Tables 4.4 and 4.5).
4.1.2.5 The Interactive Environment-Dependent Process for Matching in the Futures Market

Realistically, agents may arrive or leave this market randomly. We can therefore no longer presume a perfect collection of asks and bids during a unit of time, even employing the collective method of supply and demand. It is easy to verify that we have invalidity of the supply and demand curves in the case of a continuous double auction.

In the market system, any agent can be either a seller or a buyer. In a trade, there must be a constant quantity for supply and a given array of bids. The set of both ask and bid orders should not be changed within the unit time duration, whereas either in the batch auction or the double auction, we can presume a variable stock of
supply, because any order can be accepted within one period of time. However, this assumption leads to a variable environment around the trade in any given period. Given a variable environment, it is better to employ the master equation dynamics for solving the equilibrium of trading. It is still not known whether this solution is equivalent to an auction solution, so we must observe that we have different notions of equilibrium between constant and variable environments at a given time.

An agent may intermittently arrive at the market. The arrival rate can change, and the agent’s order is unknown, as is their mode of trading: sell, buy, or nothing. It therefore becomes important for the stock exchange system to induce all the agents to decide their trades by exhibiting a feasible price range. The way to do this is an institutional technique linked to a historical custom.

4.2 The U-Mart System and Historical Background

4.2.1 The U-Mart System Approach to the Futures Stock Market

In the final quarter of the last century, the futures market operation became a conspicuous feature of the global economy. However, many econophysicists remain interested in price formation and distribution in spot markets, partly because of limitations of data availability. The U-Mart system (Shiozawa et al. 2008) is an AI market system to implement a virtual futures market with reference to the actual stock price index arbitrarily chosen, using agent-based simulation techniques. This system contains spot market trading as a special case. Agents must finally clear trades by the spot price at the end of trade (delivery date). A hybrid approach is employed in this system in the sense that a human agent can always join in the machine agent gaming setting.\(^1\)

There are two important features of the U-Mart system. An agent, whether machine (i.e., algorithm) or human, does not presume a certain personal rational demand function in advance (Koyama 2008; LeBaron 2006). The system adopts a hybrid approach, and a human agent can always join in the machine agent gaming setting. The latter is a technological feature, a new network innovation of an AI market system, while the former is an alternative approach to the neoclassical method.

\(^1\)U-Mart started in 1998 as V-Mart (Virtual Mart), but is now called Unreal Market as an artificial research test bed. The U-Mart Project has just published an English textbook (Shiozawa et al. 2008) as one of the Springer Series on Agent-Based Social Systems. The development of the U-Mart system was mainly engineer-driven (http://www.u-mart.org/html/index.html), and is now internationally recognized as a good platform for AI markets. The project has had a policy of publicizing all program sources. Many other reports of AI market simulations provide no information about how to operate the AI. We believe that the results of market simulations by secret sources may be almost worthless.
The implementation of the *rational demand function* into the exchange system, and attempts to operate its artificial market, must not be realistic, because the market fluctuations require each agent to update or reconstruct their own demand function in each trading session. In this case, neoclassical computation should not be achieved. In the U-Mart system, therefore, any machine agent behaves according to its 'technical analysis' base. Each agent also uses a so-called technical analysis to estimate pricing on their own empirical base. Otherwise, trading may place a heavy strain on the human brain or computer.

Zero-intelligence agents (Gode and Sunder 1993) can earn in both the spot and futures market. We can easily show that the U-Mart system could be a test bed to analyze the behaviors of zero-intelligence agents in both markets. We also show why they can compete with intelligent agents to win the game, by employing a random strategy algorithm as the *arbitrage mechanism* and using position management by a trading agent. This will be explained further in Sect. 4.3.

Japanese technology was advanced enough for Japan to be the first country in the world to construct a complete futures stock market. An examination of the differences between auction methods will clarify that they are institutionally sensitive. Theoretically speaking, the double auction in the stock exchange is a kind of continuous auction, but is bound by various rules and customs. These details are carefully fulfilled in U-Mart ver.4 as shown in Fig. 4.6. The Tokyo Stock Exchange therefore uses a Japanese term, “Zaraba”, which equates more to the English term “double auction” than “continuous auction”.  

We can show that the U-Mart system can easily produce a situation where nothing happens. A small perturbation of one parameter in the strategy could generate trading even if all the agents should employ a similar strategy with slightly changed parameters. Such a perturbation may make the dealing very effective.

### 4.2.2 Historical Background to the Tokyo Stock Market

The modern Tokyo Stock Exchange (TSE) inherits all its traditional terms from the Osaka Dojima Rice Exchange (Taniguchi et al. 2008). The Osaka Dojima was the first complete futures market in the world, with trading approved by the Shogun government in 1730, over 100 years before the Chicago futures market was founded. The Osaka Dojima is recognized by many authors as the first complete futures market in the world. Although a rice exchange, it contained no rice because of limited space. Instead, traders exchanged securities called *rice stamps.*

---

2See [http://www.tse.or.jp/english/faq/list/stockprice/p_c.html](http://www.tse.or.jp/english/faq/list/stockprice/p_c.html).

3There are various kinds and qualities of rice, so there were also many types of rice stamps.
Fig. 4.6 U-Mart ver.4 GUI for the terminal trader. Note: This simulator is called the “standalone simulator”, to distinguish it from the U-Mart Market Server
The hand gestures typically employed in auctions survived until April 30, 1998, when the computer-based stock exchange was introduced.

A so-called circuit breaker, only introduced into the New York Stock Exchange after “Black Monday” in 1987, was implemented in the Dojima Exchange from the start. It is said that there were about 1,000 trading experts (brokers) working there at any time. This exchange continued to play an essential role in setting the standard rice price for the population of Japan throughout the eighteenth century (Fig. 4.7).

The two terms used for auctions in Japan, Itayose and Zaraba, are equivalent to the Western terms batch auction, a double auction on a single good to trade multiple units in sealed bid format, and double or continuous auction, an auction on a single good to trade multiple units in an open bid format.

Many economists use the term “auction” very generally, even where the market mechanism often cannot be identified as a simple auction.

4.3 The Matching Mechanisms in the U-Mart Experiment

4.3.1 The Shapes and Performances of the Market Mechanism

When referring to the Tokyo Stock Exchange, we use the terms “Itayose” or “Zaraba”, rather than “ask/bid auction” or “double auction”, because the latter
Table 4.6 Order book sample

<table>
<thead>
<tr>
<th>Ask</th>
<th>Price</th>
<th>Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>82</td>
<td>1,798</td>
<td></td>
</tr>
<tr>
<td>73</td>
<td>1,793</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>1,779</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>1,778</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>1,776</td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>1,770</td>
<td></td>
</tr>
</tbody>
</table>

Note: The restricted range given by the exchange

terms have slightly different meanings, depending on context. The double auction mechanism is designed to achieve the following priorities for trading:

1. Market order priority
2. Higher price priority (Buy); Lower price priority (Sell)
3. Preceding offer first priority (time preference)

Zaraba is an example of an institutional linked market mechanism. The trading priorities must be associated with the following institutional devices:

1. Price movement limit
2. Special bid and ask price
3. Updating the price movement limit
4. Stop high/stop low

These institutional devices can guarantee smooth price formation in the stock exchange system. The performance of the trading mechanism depends on its own institutional settings.

The limits exhibited are not always successful in accepting bids and asks within the desired range. Either special bids or asking prices are necessary if the prices exceed the limit. The exchange system must simultaneously update the price movement limit. In Table 4.6, there is no bid within the given price movement limit. The stock exchange announces a special bid and updates the limit to accept it. Without this kind of iteration based on an institutional arrangement, quotations would not be feasible. The specification of the limit may be multiple, if there is a long tail to the bid/ask distribution. The result of the contract may depend on a particular institution and rule, so the exchange system needs a refined skill to set out and manage the price movement limit.

Even contract guidance could not necessarily guarantee a normal trade if there is a rapid fluctuation of ask/bid. In this situation, the exchange system must employ the rule of Stop High/Stop Low to interrupt current trades. This breaks the circuit of the exchange.
### Table 4.7 Updating the limit

<table>
<thead>
<tr>
<th>Ask</th>
<th>Price</th>
<th>Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>82</td>
<td>1,798</td>
<td></td>
</tr>
<tr>
<td>73</td>
<td>1,793</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>1,779</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>1,778</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>1,776</td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>1,770</td>
<td></td>
</tr>
</tbody>
</table>

Note: An updating session by the exchange

### Table 4.8 A special bidding

<table>
<thead>
<tr>
<th>Ask</th>
<th>Price</th>
<th>Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>246</td>
<td>1,770</td>
<td>special</td>
</tr>
</tbody>
</table>

Note: Special bidding in an updated session

### 4.3.1.1 Institutional Customs to Set the Stepwise Ranges for Stop High/Stop Low

Price movement limits on the bid and asked prices, i.e., a range in prices, is set out by custom on the Tokyo Stock Exchange. In Table 4.7, the prices fluctuate around 1,500–2,000 yen, so the price movement limit is taken within 20 yen, from 1,773 to 1,793. This is used to set the price contracted at the end of the previous day as the marker price for each new day. Based on the marker price, the rule of stop high and low regulates any violent price variation by stopping the trade (Table 4.8).

In 2012, TSE revised the stepwise ranges as shown in Table 4.9.

### 4.3.1.2 Visualizing the Differences in Matching Mechanisms

See Fig. 4.8; Tables 4.10 and 4.11.

### 4.3.2 Zero-Intelligence Tests in the U-Mart System

The U-Mart system is mainly designed for futures market trading. In this trading, either machine or human agents join to create a sequence of successive futures prices, with reference to given real spot market prices arbitrarily chosen. The settlement must be done at the final delivery date by employing the real spot price. This provides some entertaining gambles. By removing regulation from futures market trading, we can imitate a trade for the spot market. In other words, the spot market can be derived as a special case of the U-Mart system.
### Table 4.9 Partial revision of daily price limits and special renewal price intervals

<table>
<thead>
<tr>
<th>Price (JPY)</th>
<th>Daily price limits</th>
<th>Renewal price intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Revised</td>
</tr>
<tr>
<td>~100</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>100–200</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>200–500</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>500–700</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>700–1,000</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>1,000–1,500</td>
<td>200</td>
<td>300</td>
</tr>
<tr>
<td>1,500–2,000</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>2,000–3,000</td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>3,000–5,000</td>
<td>500</td>
<td>700</td>
</tr>
<tr>
<td>5,000–7,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>7,000–10,000</td>
<td>1,000</td>
<td>1,500</td>
</tr>
<tr>
<td>10,000–15,000</td>
<td>2,000</td>
<td>3,000</td>
</tr>
<tr>
<td>15,000–20,000</td>
<td>2,000</td>
<td>4,000</td>
</tr>
<tr>
<td>20,000–30,000</td>
<td>3,000</td>
<td>5,000</td>
</tr>
<tr>
<td>30,000–50,000</td>
<td>4,000</td>
<td>7,000</td>
</tr>
<tr>
<td>50,000–70,000</td>
<td>5,000</td>
<td>10,000</td>
</tr>
<tr>
<td>70,000–100,000</td>
<td>10,000</td>
<td>15,000</td>
</tr>
<tr>
<td>100,000–150,000</td>
<td>20,000</td>
<td>30,000</td>
</tr>
<tr>
<td>150,000–200,000</td>
<td>30,000</td>
<td>40,000</td>
</tr>
<tr>
<td>300,000–500,000</td>
<td>50,000</td>
<td>70,000</td>
</tr>
<tr>
<td>500,000–700,000</td>
<td>100,000</td>
<td>100,000</td>
</tr>
<tr>
<td>700,000–1,000,000</td>
<td>100,000</td>
<td>150,000</td>
</tr>
</tbody>
</table>

Note: The highlighted cells indicate newly revised prices

---

**Fig. 4.8** Left, partial revision of daily price limits and special renewal price intervals; right, Itayose matching
Table 4.10  The first scene

<table>
<thead>
<tr>
<th>MO</th>
<th>1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,000</td>
<td>1,702</td>
</tr>
<tr>
<td>8,000</td>
<td>1,701</td>
</tr>
<tr>
<td>1,700</td>
<td>4,000</td>
</tr>
<tr>
<td>1,699</td>
<td>7,000</td>
</tr>
<tr>
<td>1,698</td>
<td>20,000</td>
</tr>
</tbody>
</table>

Table 4.11  The second scene

<table>
<thead>
<tr>
<th>MO</th>
<th>1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,000</td>
<td>1,702</td>
</tr>
<tr>
<td>7,000</td>
<td>1,701</td>
</tr>
<tr>
<td>1,700</td>
<td>4,000</td>
</tr>
<tr>
<td>1,699</td>
<td>7,000</td>
</tr>
<tr>
<td>6,000</td>
<td>1,698</td>
</tr>
</tbody>
</table>

In the U-Mart system, default loaded agents with technical analyses are:

TrendStrategy: Set price1=last futures price, and price2=second last futures price. If price1 is higher than price2 then the agent orders buying. If price1 is lower than price2, then the agent orders sell. The amount of order is randomly decided.

AntiTrendStrategy: price1=last futures price, and price2=second last futures price. If price1 is lower than price2 then the agent orders buying. If price1 is higher than price2 then the agent orders sell. The amount of order is randomly decided.

RandomStrategy: The agent buys or sells randomly. The limited price on order is set randomly around the latest futures price, and quantity of the order is set randomly within a prescribed range. Position of the agent is also considered in decision making.

SRandomStrategy: The agent buys or sells randomly. The limited price on order is set randomly around the latest “spot price”, and quantity of the order is set randomly within a prescribed range. The position of the agent is also considered in decision making.

RsiStrategy (RSI: Relative Strength Index): The agent buys or sells according to the Relative Strength Index (RSI) of futures price. RSI is one of major technical analysis methods. The limited price is given randomly around the latest futures price, and quantity of the order is given randomly within a prescribed range. The position of the agent is also considered in decision making.

---

4The U-Mart Project publicizes the fundamental default strategies on the site: http://www.u-mart.org/html/contentsE/sampleagentE.html. The copyrights of default strategies belong to (c)2000 Rikiya FUKUMOTO (c)2002 U-Mart Project.

5In the following, the first capital letter “S” means “spot prices”.

SRsiStrategy: The agent buys or sells based on Relative Strength Index (RSI) of spot price. RSI is one of major technical analysis methods. The limited price is set randomly around the latest spot price, and quantity of the order is set randomly within a prescribed range. The position of the agent is also considered in decision making.

MovingAverageStrategy: The agent sends an order when the short term moving average line of futures price crosses over the medium term moving average line. If the trend of the short term moving average line is up, the agent gives buy order and when it is down, he gives sell order.

SMovingAverageStrategy: The agent sends an order when the short term moving average line of the spot price crosses over the medium term moving average line. If the trend of the short term moving average line is up, the agent gives buy order and when it is down, he gives sell order.

SFSpreadStrategy: The agent orders when a spread between the spot and then future price is wider than threshold. If the future price is higher than the spot price, the agent gives buy order, and if the future price is lower than the spot price, he gives sell order.

DayTradeStrategy: This is the kind of day trading strategy. The agent gives sell and buy orders simultaneously. The limit price of sell order is slightly higher than latest futures price. The limit price of buy order is lower than latest futures price.

In the latest version of the U-Mart system called U-Mart ver.4, the default agent set has been slightly expanded to incorporate MarketRandomStrategy. It is also noted that the names of agents based on the future prices are changed from TrendStrategy into UTrendStrategy, AntiTrendStrategy into UAntiTrendStrategy, RSiStrategy into URsiStrategy, Moving AverageStrategy into UMovingAverageStrategy.

Needless to say, a mixture of several of these strategies may be feasible.

One of the default settings is the random strategy agent. This means employing simultaneous random moves on mode choice (sell or buy) and limit order (price and quantity). The choice entirely depends on pseudo-random number generation by a computer. We can therefore use these as zero-intelligence agents, and conduct two kinds of zero-intelligence simulations: one for the futures market, the other for the spot market.

Over the long history of the U-Mart experiment, we have become familiar with the workings of the random strategy in the futures market. We have developed an empirical rule that the random strategy is not defeated by many other strategies, and it may even be a winning strategy when all other agents are similar. This experiment is quite easily run by a standalone simulator (Fig. 4.6), and I recommend trying it to expand your repertoire of market simulation and discover new heuristics in market trade properties. We use this strategy as a test for new machines.
4.4 Similarities of Trading Strategies Between SF Spread and Random Strategy

4.4.1 Arbitrage: Equalization Between Markets

Profit-seeking activities will make different situations more similar. This is called “arbitrage”:

- If the spot price is higher than the futures price, then buy futures and simultaneously sell the same amount in the spot market.
- If the position of both markets is kept fixed until the settlement date, very little loss will be incurred, because settlement is conducted using the same price (the spot price) to the same positions. If the spot price falls rapidly (spot price $\gg$ future price, or the SF spread is too large), profits will be made.

4.4.1.1 The Essence of Arbitrage

Arbitrage is one of the most important mechanisms to intermediate different kinds of market. Here, we focus on arbitrage between the spot and futures markets, where it may be defined as the management of the short and long positions, which is effectively risk management on trading.

Historically, when he commented on Hayek’s idea of the natural rate of interest, Sraffa (1932a, b) using arbitrage, cleverly explored how there was no such unique rate in the market. Sraffa’s idea is easily illustrated using the order book. Sraffa considered the trade in bales of raw cotton, but only the so-called “day trader’s method”, the implementation of simultaneous alternative moves, i.e., cross trades of ask in the spot market and bid in the futures market, or bid in the spot market and ask in the futures market. The next numerical example uses the cross trades of ask in the spot market and bid in the futures market:

In this case, settlement in both markets could be feasible if $x$ was implemented. In this case, the concerned trader can earn ¥10,000(= 1,100 − 1,000 × 100 bales). The self-interest rate is therefore calculated as (Tables 4.12 and 4.13):

$$r_s^c = \frac{1,100 - 1,000}{1,000}$$  \hspace{1cm} (4.3)

<table>
<thead>
<tr>
<th>Table 4.12</th>
<th>The spot market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ask</td>
<td>Price</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4.13</th>
<th>The futures market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ask</td>
<td>Price</td>
</tr>
<tr>
<td>$x$</td>
<td>1,100</td>
</tr>
</tbody>
</table>

6Given a single market, this may be simultaneous ask and bid.
The rate of interest is a personal rate, which is connected with an intertemporal rate common to the traders whose operation can be established directly in the market. Sraffa called this “the commodity (cotton) rate of interest”. This definition can be generalized to many other commodities with their own market both for spot trades and futures. It is also clear that there are as many commodity rates of interest as there are commodities.

Sraffa criticized Hayek’s naive opinion that a single natural rate of interest could exist. Hayek was not successful in formulating a credit system. He did not understand that there were multiple commodity rates of interest, or how such multiple rates could be equalized. The equalization never holds automatically. Implementing it requires a particular mechanism of credit allocation to appropriate investment funds for traders, as well as a particular institutional deployment. We have already learned that the market mechanism always underlies institutional formation. Without understanding the effects of the current institutional framework, no innate force can emerge, as Hayek imagined for the natural rate of interest.

We can compare Sraffa’s objective definition with Fisher’s subjective one:

Irving Fisher carried out work in the 1930s to define the ‘own’ rate of interest. He intended to derive it directly by the subjective motivation of consumption. The ‘own’ rate of interest of an apple for a consumer is interpreted as the exchange rate of today’s apple $a_t$ with tomorrow’s apple $a_{t+1}$. By this definition, on the surface, the subjective rate of temporal exchange seems to hold, as does the idea that tomorrow’s apple should be appreciated more than today’s because it has been earned by giving up a part of today’s. The difference between $a_t$ and $a_{t+1}$ may then be expected to be the sacrifice equivalence for the consumer of today, to reserve it for tomorrow. The subject’s inner preference therefore decides the ‘own’ rate of interest. This also seems to support the next relation:

$$ a_t \leq a_{t+1}. \quad (4.4) $$

However, the converse relation never gives a natural meaning of patience. It rather refers to wasting. We therefore need to force the former relation to be held by introducing the evaluation of the discounted price system. The discounted price, by definition, is always decreasing over time. If we employ the current prices $P_t$, then the number of apples purchased by ¥1 at date $t$, i.e., $a_t$, and the number of apples purchased by ¥1 at date $t+1$, i.e., $a_{t+1}$, are defined as:

---

7See Fisher (1930, 1974).
If the apples are purchased by borrowing money at the money rate of interest $r_m$, it then holds that:

$$\frac{(1 + r_m)(P_t - P_{t+1})}{P_{t+1}} = r_a$$  \hspace{1cm} (4.6)

It is then trivial that the ‘own’ rate of apples $r_a$ may be negative if the part $P_t - P_{t+1}$ of the numerator is negative.

If the market works freely, the subjective meaning of the intertemporal rate could be lost. This method can neither stand by itself nor adapt to the market. The meaning of the own rate of interest does not cause any difficulty, even when negative, if Sraffa’s method is employed. This is a natural outcome of the market mechanism. As the market fluctuates, the rate of exchange also changes in terms of price. The price may be down tomorrow, and then the own rate must be negative.

In the past, economists often insisted that there was an easy connectivity between intertemporal choices between the consumption and transformation sets of production, to give a kind of intertemporal equilibrium. However, this connection will never be generated by the arbitrage mechanism working through the markets, and this thinking results in a false sense of reality.

As stated at the beginning of this section, the function of arbitrage in the spot and futures markets will imply the need for position management for the traders. Arbitrage of this kind can also cause discrepancies between multiple commodity rates of interest, whether positive or negative. We now consider the working of arbitrage in a special example.

### 4.4.1.2 SF Spread Strategy and Arbitrage

SF spread represents the distance between the spot price and the futures price. We now consider arbitrage between the spot and futures markets. The agent orders when a spread between spot and future price is wider than the threshold. If the future price is higher, the agent gives a buying order, and if it is lower, a selling order (Fig. 4.9).

The order price is given by a Gaussian distribution whose mean is set as the latest future price $p^f[t]$ and whose standard deviation is set as the price change band triangle. Order volume is a uniform random number between min $q^f$ and max $q^f$. 
4.4 Similarities of Trading Strategies Between SF Spread and Random Strategy

4.4.1.3 Random Strategy

The agent buys or sells randomly. The limited price on order is set randomly around the latest price, and quantity is set randomly within a prescribed range. Either position of the agent is also limited to the upper limit.

SRandom: employ the latest spot price  
FRandom: employ the latest futures price

If the random strategy is not successful in any particular session, the order is held over to the next session, and kept as a position in either direction (short or long). The sessions therefore accumulate the position until the end of the day, when the mark to the market is executed. Each day will have either a short or long position. On average during the whole trading period, the positions will be canceled out (Fig. 4.10).
4.4.1.4 Similarities Between SF Spread and Random strategies

The agent decides whether to buy[0], sell[1], or donothing[2] according to 0, 1, 2 randomly generated. No order is given if $q^f[t + 1]$ generates a result exceeding the given prescribed position. The size of the short and long position is limited.

$$p^f[t + 1] = p^f[t] + \text{the constant price change band} \Delta$$

$\times$ random real number generated by a Gaussian distribution

$$q^f[t + 1] = \min q^f + \text{random integer within the range of } (\min q^f, \max q^f)$$

(4.7)

If

$$\frac{p^f[t] - p^s[t]}{p^s[t]} \geq \bar{\nu}, \text{ sell futures } q^f[t + 1]$$

(4.8)

This inequality is then equivalent to:

$$p^f[t] \geq (1 - \bar{\nu}) p^s[t].$$

(4.9)

Here $\bar{\nu}$ is an arbitrary threshold.

We can define the ‘buy’ condition as:

If

$$\frac{p^f[t] - p^s[t]}{p^s[t]} \leq \bar{\nu}, \text{ buy futures } q^f[t + 1]$$

(4.10)

This inequality is then equivalent to:

$$(1 - \bar{\nu}) p^s[t] \geq \hat{f} [t].$$

(4.11)

Here $\bar{\nu}$ is also an arbitrary threshold.

In this market system of spot and futures, arbitrage between the spot and futures markets may be defined as:

If $p^f[t] \geq p^s[t], \text{ sell futures, and buy the same orders } q^f[t + 1]$ in the spot.

(4.12)

---

8This part depends on Kita (2012). Prof. Hajime Kita, Kyoto University, has recognized this fact and arranged well the U-Mart Project of a simple shaped market at the beginning of this project. The description of this subsection depends on his discussion.
4.4 Similarities of Trading Strategies Between SF Spread and Random Strategy

If \( p_f^t[t] \leq p_s^t[0] \), buy futures, and sell the same orders \( q_s^t[t + 1] \) in the spot.

(4.13)

These numerators exactly correspond to the conditions for arbitrage. It is therefore clear that the SF spread strategy is essentially equivalent to arbitrage between the spot and futures market.

4.4.2 The Performance of the Random Agents in U-Mart ver. 4’s Simulation

U-Mart deals with the futures trade, at present, by taking into account spot market motion. Usually, for the convenience of traders, the spot market price data are extrapolated into the GUI screen. We arrange technical agents of both types, whether the reference price series used to forecast the future market motion is that of spot (S type) or future prices (U type). Here, we consider two kinds of experimental design to examine the effects of random agents. Random agents in U-Mart are classified into two types: \textbf{SRandomStrategy}, which randomly gives orders around the spot price series, and \textbf{URandomStrategy}, which gives orders around the futures price series. We use the default setting for experiments, with two sessions per day over 5 days, depending on the TSE session time allocation in Fig. 4.11. We also use the same spot price series chosen in the default data set.\(^9\)

**Case 1 experiment** We use a set of technical agents: URandomStrategy (5 units), SRandomStrategy (5 units), UTrendStrategy (1 unit), STrendStrategy (1 unit), UAntiTrendStrategy (1 units), SAntiTrendStrategy (5 units), URsiStrategy (1 units), SRsiStrategy (1 units), and UMovingAverageStrategy (1 units), UMAovingAverageStrategy (1 units).

**Case 2 experiment** Here, we limit the agent types to a random strategy: URandomStrategy (5 units) and SRandomStrategy (5 units).

Fig. 4.11 TSE ’s session time allocation

---

\(^9\)Our spot time series is adapted from ‘2009-5-25_2010-1-6.csv’ in src/nativeConfig in the U-Mart ver. 4 system.
4.4.2.1 Case 1: Including Technical Agents Apart from Random Ones

In our custom, the trade volume in the futures market is in the plane measured by the vertical axes on the right-hand side. The peak values of transaction repeated alternatively from each cycle of 120 to each 150 units of time. These figures just are established by the TSE session time allocation, shown in Fig. 4.11.

Our results reveal a good performance of SRandom in comparison with URandom strategies. This may reflect the situation when the futures price series is generated around the spot price one. If this were the case, the probability of stipulation would be increased. A higher rate of stipulation does not necessarily guarantee profit, but may increase the changes of a profit in the Zaraba or double auction. When the futures price moves more than the spot price series, the agent working on the basis of the futures price may not only have less chance to make profit, but also suffer a higher probability of being matched by others at a very adverse rate. In the U-Mart experiments, then, the SRandom strategy will usually achieve a much better performance than the URandom one.

We already noted that both cases had the same spot market implementation for both simulations. In both cases, the SRandom group performed better than the URandom one. This shows that the average profit of the SRandom strategy group is much higher than that of the URandom strategy group. All the URandom agents, but only one SRandom agent, recorded negative profits (Fig. 4.12).

4.4.2.2 Case 2: A Comparison of Performance Between SRandom and URandom

In Case 2, if we examine the position management for each agent, SRandom agent 1, who suffered from a consequential loss, behaved as if there was no intelligent position management at all. The cross-section between total buy and sell, which is equivalent to a loss-cut, appear about eight times. The loss to SRandom agent 1 is estimated to be less. Similar reasoning can be applied to URandom agent 1 is estimated to be less. Similar reasoning can be applied to URandom agent 4, who suffered least, and to SRandom agent 3 and URandom agent 5 in Case 1 (Figs. 4.13 and 4.14).

We suggest that SRandom agents like 2 and 4, who are committed to position management, secured big profits because at the settlement date in this experiment, the spot price exceeded the futures price. The situation therefore favors those agents whose total buy is greater than their total sell. They can then resell the excess buy at a higher spot price to confirm the excess profit.
Fig. 4.12  Case 1 Experiment. Note: The highlighted band around the spot price line represents ‘Bollinger band’, which is a ‘technical analysis tool’ invented by John Bollinger in the 1980s
Fig. 4.13 Case 2 Experiment
Fig. 4.14 Case 1 Position managements of random strategies
References

Fisher I (1930, 1974) Theory of interest. Augstus Kelley, Clifton
Sraffa P (1932a) Dr. Hayek on money and capital. Econ J 42:42–53
Sraffa P (1932b) A rejoinder. Econ J 42:249–251
Chapter 5
The Evolution of the Market and Its Growing Complexity

Abstract  The market is evolving, and particularly remarkably recently. Algorithm dominance is rapidly developing. Algorithm-based evolution is not limited to the capability of exchange servers, and humans are being replaced by transaction algorithms. The shift to high-frequency trading (HFT) has explicitly revealed the discrepancy between logical and practical transaction times. We no longer work in the realms of logical time. A slight difference in transaction speed or server design will affect the market results. We are also entering a new stage of surveillance because of the risk of illegal operations by traders. A brief look at market evolution shows a massive change in human relationships. Not only does developing algorithm dominance have an influence, but also the growing complexity, which gives rise to “the rich getting richer”. Standard economics, however, seems quite oblivious to these changes.

5.1 Practical and Logical Time in High-Frequency Trading (HFT): A Re-domaining of the Trading System

Traditional economists focused on game-theoretic auctions on a logical time basis. Now that algorithmic trade is dominant, it seems strange to observe game-theoretic behaviors by focusing only on abstract ‘logical’ time. In the current financial market, algorithmic trading has become dominant, and orders are achieved each microsecond, although the stock exchange’s server can only process them within milliseconds. High-frequency trades (HFTs) are characterized by the delay between the process speed and the order speed. If algorithmic trade and server-based exchanges are both essential, this kind of delay must be inevitable. We suggest that this should be seen as a re-domaining of the trading system, away from auctions on a logical time basis. We need algorithmic trade to progress.

Section 5.4 first appeared in Aruka and Akiyama (2009)
5.1.1 Caveats on HFT from the European Securities and Markets Authority

Without any caveats on HFT, we will inevitably see deceptive behaviors. The European Securities and Markets Authority (ESMA) therefore issued a consultation paper (ESMA 2011) setting out the four major deceptive behaviors often seen:

- Ping orders, making orders for small amounts to trigger reactions in other participants, bringing additional information about their positions and expectations; Quote stuffing, making small variations in the position in the order book to create uncertainty for other participants, slow down the process and hide their own strategy;
- Momentum ignition, making aggressive orders to start or exacerbate a trend, hoping others will follow and help to unwind the position;
- Layering and spoofing, submitting multiple orders at different prices on one side of the order book, submitting a single order to the other side of the order book (which reflects the true intention to trade) and, once that has been executed, rapidly removing the multiple initial orders from the book.

All these strategies are well known, but were generally suppressed before HFT, because they are illegal or punishable. As long as any order can be offered and immediately canceled out within the unit time process of the exchange server, these behaviors are possible. Layering and spoofing is usually a cross-operation. In the next section, we show how it is done.

5.1.1.1 Deceptive Behaviors and Cross-Operation by an HFT

As set out in Chap. 4, the stock exchange operates a continuous double auction governed by rules and institutional settings. The advent of HFT, however, brought a new hierarchical structure, forcing a re-domaining of the market form. The market can create a so-called two-fold layer, consisting of the outer process and the inner field. In the latter, algorithm traders make their own moves within the unit process time of the server, irrespective of any regulation of the auction rules. This stage may be considered a form of unregulated double auction. The hierarchical structure created by the time delay implies that some illegal moves in the double auction system could be possible but invisible, creating a second invisible market with the HFT system.

By exploiting the time delay in the processing of matching it has become feasible to execute apparently illegal orders like cross-operations without being detected. We now illustrate a possible case using a cross-operation, where an ask and bid are fabricated far from the real level. This is only possible if the order speed outstrips that of processing, which is the case on the stock exchange.
Table 5.1 Cross-operational move by agent F

<table>
<thead>
<tr>
<th>MO for sell</th>
<th>Ask</th>
<th>Price</th>
<th>Bid</th>
<th>MO for buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>F3,000</td>
<td>100</td>
<td>F1,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>99</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>96</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1,000</td>
<td>92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H1,500</td>
<td>91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>A3,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2 Induced move by agent Z

<table>
<thead>
<tr>
<th>MO for sell</th>
<th>Ask</th>
<th>Price</th>
<th>Bid</th>
<th>MO for buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>F3,000</td>
<td>100</td>
<td>F1,000+Z2,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>99</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>96</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1,000</td>
<td>92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H1,500</td>
<td>91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>A3,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.1.1.2 Stage 1

Agent F makes a cross-operational move although the real supply and demand tends to be fixed around ¥90 to 92 (see Table 5.1). Agent Z may be induced by agent F’s actions to have a limit order of 2,000 units at ¥100 (see Table 5.2).

5.1.1.3 Stage 2

After agent Z’s move, agent F may immediately make the next move and ask for 2,000 units by market order. This move will drastically change the market situation, so agent Z is forced to cut his losses by submitting his ask of 2,000 units in the form of a limit order at ¥90 to improve his position balance and cancel out the ‘wrong’ order posted at ¥100 (see Table 5.3).

If agent Z confirms his successful ask at ¥90, the contract would make the initial effort to make the cross-operation futile. The price level is what was expected from the real supply and demand, and agent F is therefore keen to ensure that his next move leads to the desired result.
### Table 5.3 The next moves

<table>
<thead>
<tr>
<th>MO for sell</th>
<th>Ask (sell)</th>
<th>Price</th>
<th>Bid (buy)</th>
<th>MO for buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>F2,000</td>
<td>F3,000</td>
<td>100</td>
<td>F1,000 +</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Z2,000</td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td></td>
<td>99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td></td>
<td>98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td></td>
<td>97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td></td>
<td>96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td></td>
<td>95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td></td>
<td>94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td></td>
<td>93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td>G1,000</td>
<td>92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td>H1,500</td>
<td>91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td>Z2,000losscut</td>
<td>90</td>
<td>A3,000</td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td>‡</td>
<td>89</td>
<td>F1,000$\text{show}_i$</td>
<td>$\leftarrow$ cancelled</td>
</tr>
<tr>
<td>F2,000</td>
<td>‡</td>
<td>88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td>‡</td>
<td>87</td>
<td>F2,000$\text{show}_i$</td>
<td>$\leftarrow$ cancelled</td>
</tr>
<tr>
<td>F2,000</td>
<td>‡</td>
<td>86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td>‡</td>
<td>85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td>‡</td>
<td>84</td>
<td>F3,000$\text{show}_i$</td>
<td>$\leftarrow$ cancelled</td>
</tr>
<tr>
<td>F2,000</td>
<td>‡</td>
<td>83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td>‡</td>
<td>82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td>F1,000special</td>
<td>81</td>
<td>3,000</td>
<td></td>
</tr>
<tr>
<td>F2,000</td>
<td></td>
<td>80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Orders by agent F are immediately canceled when agent Z cuts their losses.

### Table 5.4 The position of agent F

<table>
<thead>
<tr>
<th>Time</th>
<th>Sell balance</th>
<th>Buy balance</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,000</td>
<td>1,000</td>
<td>2,000 short</td>
</tr>
<tr>
<td>⋮</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>2,000</td>
<td>2,000</td>
<td>0</td>
</tr>
<tr>
<td>$t + 1$</td>
<td>4,000</td>
<td>4,000</td>
<td>0</td>
</tr>
</tbody>
</table>

#### 5.1.1.4 Stage 3

Given the actions of agent Z, agent F will at once make a countermove to deploy show bid limit orders like 1,000 units at $\{¥89\}$, 2,000 units at $\{¥87\}$, 3,000 units at $\{¥84\}$, or 3,000 units at $\{¥81\}$. These show bid orders, except for the last one of 3,000 units at $\{¥81\}$, which will be canceled soon after a special ask of 1,000 units at $\{¥81\}$. In the final stage, agent F will confirm the end by short-selling the limit order of 1,000 units with a large price fall, to cancel out the other bids. The bigger the price fall, the greater the profit, so agent F will win the game. In this game, the first ask of 3,000 units at 100 will be purchased by the final special bid (see Table 5.3). Tables 5.4 and 5.5 illustrate the position balance of the two agents. The only requisite for success is to move as quickly as possible, which is the operating principle for HFT.
5.1 Practical and Logical Time in High-Frequency Trading (HFT)

Table 5.5 The position of Z

<table>
<thead>
<tr>
<th>Time</th>
<th>Sell balance</th>
<th>Buy balance</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2,000</td>
<td>2,000 long</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>2,000 move</td>
<td>2,000</td>
<td>blocked by F</td>
</tr>
<tr>
<td>$t + 1$</td>
<td>0</td>
<td>2,000</td>
<td>2,000 long</td>
</tr>
</tbody>
</table>

5.1.2 The Re-domaining of the Market Caused by the HTF System

We have now seen the process of an unregulated double auction as compatible with a cross-operation by a speculator, although such an operation is an illegal action in the stock exchange. Given the delay of process speed behind order speed, a two-fold hierarchy has been created, with the familiar double auction as the outer system and an invisible mechanism embedded within it (see Fig. 5.1). The two systems interact, and work together as a new system separated from the supposed natural system of logical time processing. In the algorithmic world, what matters is the practical time required to process-match and submit orders, and the difference between the two can no longer be ignored.

Before the development of algorithmic trading, there was a clear distinction between batch and double auctions, as we discussed in Chap. 4. Reconstructing these by algorithm, a process of matching will take place in each time unit. The more precision required, the smaller the unit of time must be, but it is impossible to secure the ideal of continuous time. The double auction system in algorithmic space will therefore take a new form, shaping a two-fold hierarchy for trading. The evolution of the market will depend on the interaction between the outer and inner systems. See Fig. 5.2. The visible layer just shows a part of the evolution at the hidden layer. See Fig. 5.3.

The upper panel of Fig. 5.4 shows the difference between batch and double auctions in terms of unit time structure, and the lower panel of it shows the two-fold hierarchy’s unit time structure. The comparison between the traditional and algorithmic trading systems is shown in the two arrows depicted in Fig. 5.4. Here the first pivot (unit of time) and the second pivot in the double auction can be furthermore decomposed into the number of unit time.

5.1.3 A Historical Example: A Flash Crash

In a natural system, the patterns of a movement will change dramatically with a change in its speed. Mainzer (2007) summarized this proposition by taking the instance of the pattern of a stream. The velocity of the stream’s current changes its flow pattern. As the velocity of a stream increases:
Fig. 5.1  The invisible layer imbedded in the trading system shown in two dimensions

Fig. 5.2  The evolution of the hidden layer

Fig. 5.3  The visible layer in the HTF
5.1 Practical and Logical Time in High-Frequency Trading (HFT)...

**Fig. 5.4** The hierarchical trading system

<table>
<thead>
<tr>
<th>Rule</th>
<th>Session time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batch auction</td>
<td>![Batch auction]</td>
</tr>
<tr>
<td>Double auction</td>
<td>![Double auction]</td>
</tr>
<tr>
<td>Field</td>
<td>![Field]</td>
</tr>
<tr>
<td>Visible transaction</td>
<td>![Visible transaction]</td>
</tr>
<tr>
<td>Hidden transaction</td>
<td>![Hidden transaction]</td>
</tr>
<tr>
<td>HFT</td>
<td>milli/micro second</td>
</tr>
</tbody>
</table>

**Fig. 5.5** Pattern variations: flow around a sphere at different velocities. Speed (Reynolds number) 20; 100; 150; 250; [http://demonstrations.wolfram.com/FlowAroundASphereAtFinite-ReynoldsNumberByGalerkinMethod/](http://demonstrations.wolfram.com/FlowAroundASphereAtFinite-ReynoldsNumberByGalerkinMethod/)

1. Its periodicity will change from regular to irregular; and
2. The pattern of the current will become increasingly complex.

The current financial system is subject to both algorithmic trading and HTF. The speed of trading is likely to cause various effects, such as the 2010 “flash crash”. Though alarming at the time, this may be a natural outcome of the various patterns emerging from a nonlinear system, and only one possible outcome of the hyper-velocity and frequency of stock trading (Fig. 5.5).
Before considering the nonlinear properties of the system in more detail, we first give an outline of the flash crash (Fig. 5.6):

On May 6, US stock markets opened down and trended down most of the day on worries about the debt crisis in Greece. At 2:42 pm, with the Dow Jones down more than 300 points for the day, the equity market began to fall rapidly, dropping an additional 600 points in 5 minutes for an almost 1,000 point loss on the day by 2:47 pm. Twenty minutes later, by 3:07 pm, the market had regained most of the 600 point drop.

The financial markets had shown the following properties around the HFT system:

High frequency or high oscillation of the system will alter its phase.
High-speed matching or high-speed processing of orders will cause the system to oscillate.
Even under the same matching rule, the outcome will differ qualitatively when the matching system enters hyper-speed processing.

Based on knowledge of nonlinear sciences, we can suggest suitable regulatory ingredients. First, we need to know the essential features in a given system:

1. The institutional setting and rules
2. What kind of system is it designed to be?

---

3. Characteristics (important factors to be considered):

- What kind of landscape: configuration?
- Which direction?
- High frequencies

4. Systemic risk of the movement

5. Fragmentation and instability

The essential features of the economy and the market system are:

1. The *free* market system: system operation
2. The *free* market system: agent strategies
3. Characteristics
   - Large number of participants
   - Very high speed of processing

4. Algorithmic intelligence dominance

5. Social fragmentation and its rapid developments (Fig. 5.7)

In the financial markets, algorithmic intelligence has come to dominate:

The Carnivore, the Boston Shuffler, the Knife are some of the names of the algorithms on Wall Street that make up 70% of the operating system of stocks formerly known as your pension fund, and portfolio. What Kevin Slavin is talking about is High Frequency Trading/Black Boxes that dominate The Street. In reality these mathematic algorithms are shaping far more than Wall Street, they’re shaping our world.²

---

**Fig. 5.7** Notional principal value of outstanding derivative contracts, as recorded at year end. These include foreign exchange, interest rates, equities, commodities and credit derivatives. Data from UK Department for Business, Innovation and Skills, International Monetary Fund and Bank of England calculations. Hundreds of trillions balance of the derivative contracts. Cited from Fig. 1 in Haldane and May (2011)

²Jozwicki (2011).
5.1.4 How a Flash Crash Happened

5.1.4.1 The Triggering Agents

There were some important causative moments leading up to the flash crash. A large number of ‘asks’ could be enough to freeze the bid orders, and lead to a ‘price plunge’ until the price contract is nearly zero, which is what happened in the May 2010 flash crash. Even if a single order was not quite large enough to deplete all the bid orders, traders can panic if the price of a security goes down rapidly. They then add more ask orders, reinforcing the situation and resulting in a flash crash (Misra and Bar-Yam 2011).

The triggered algorithmic agents were considered as these rather simple-shaped agents:

Arithmetic limit order: raise or lower 1 by 1 cent per 1/1,000 second
Geometric limit order: raise or lower double per 1/1,000 second

The synchronization between them contributed to a sudden rapid fall in the stock price, achieving almost zero pricing instantly. In the current market, there is an unprecedentedly large transaction balance created by the movement of various financial commodities as well as large numbers of algorithmic agents. The flash crash followed a much larger crash, linked to the collapse of Lehman Brothers. After this flash crash in 2010, there were several others too, but these caused little external damage because large unilateral asks could be canceled out by a bid the same size. This may be feasible by a single agent who has simultaneous moves of ask and bid. The occurrence probability of shocks is often said to be similar to a large earthquake. However, small shocks have occurred thousands of times a year since May 2010, and there are likely to be many more as hyper-speed trading expands.

The financial market therefore seems to be less stable. A microsecond in the markets can be as crucial as the microsecond needed to press the nuclear missile button. Recent studies have often shown the market taking a random walk that does not necessarily follow the central limit theorem. One particular trader can deliberately change the market configuration, and cause an upward then downward surge, adding instability. A quick injection of bid action will guarantee a quick recovery. If such instability is embedded, a violent fluctuation could offer huge profits, so price plunges may be welcomed by traders.

Remark 5.1. HM The Queen asked mainstream economists why the Lehman shock had happened. A group of distinguished economists from the London School of Economics and the Royal Economic Society replied:

So where was the problem? Everyone seemed to be doing their own job properly on its own merit. And according to standard measures of success, they were often doing it well. The failure was to see how collectively this added up to a series of interconnected imbalances over which no single authority had jurisdiction. This, combined with the psychology of herding and the mantra of financial and policy gurus, led to a dangerous recipe. Individual risks may rightly have been viewed as small, but the risk to the system as a whole was vast.
So in summary, Your Majesty, the failure to foresee the timing, extent and severity of the crisis and to head it off, while it had many causes, was principally a failure of the collective imagination of many bright people, both in this country and internationally, to understand the risks to the system as a whole. (Besley and Hennessy 2009).

We now explore how this happened (Fig. 5.8).
5.2 A Stealth Market

In the last century, the idea of the invisible hand was repeated too often. The idea emerged of the market becoming transparent i.e., visible to all participants. However, the real market is never transparent. As the power law works, the environment is undergoing modifications everywhere. Instead of the traditional equilibrium price, equilibrium may be regarded as convergence towards a stable distribution like the Pareto-Levy distribution. This is a decisive difference between traditional economics and econophysics.

5.2.1 The Invisible Organization of Finance

There are some decisive institutional settings in the stealth market. The securitization of property rights including loans is a system of reassurance, which in itself is an old technique dating back to feudal times. It may be virtually a debt bond swap, making the system even more complex. The financial commodities issued initially by investment partnerships are invisible to the general public. Financial intermediation thus becomes deeper and more complex. We therefore introduce some basic ideas from complex science, such as logical depth, to support analysis.

5.2.1.1 Devastating Financial Forces Equivalent to Large Earthquakes

The similarities between financial crises and large earthquakes are often discussed, and there are even some econophysicists who were formerly seismologists. For example, the sources or causes are seldom fully explored. The widespread nature of the effects are often unseen. The same frequencies and magnitudes do not necessarily cause the same effects. The initial shock is sometimes accompanied by a second disaster. Aseismic measures are required. Both earthquakes and financial crises are often seen as very difficult to predict in both size and location. In recent crises, the expansion of the shadow banking system (SBS) is seen as significant (Issing et al. 2012). The investment partnership issuing the debt bond swap at the initial stage will unilaterally dominate the information around its reassurance. The regulator can favor a particular group or type of investors. These effects are in many cases achieved behind the scenes, so that the public do not know about the risks around these financial commodities.

5.2.1.2 Risk Management of the Classical Futures Market

In the classical financial market, double-check management in the futures market is always assured by two rules:
Mark to the market: The sessions accumulate the position until the end of the day. The mark to the market settles the balances between participants every night as if the day looked like the final contract. Margin requirement: The futures exchange institution must act as intermediary and minimize the risk of default by either party. The exchange therefore requires both parties to put up an initial amount of cash, the margin or margin call.

These rules were fully implemented in the Japanese futures market from its inception in 1730. However, the options market did not impose the rule of “mark to the market”, and therefore lost sight of the current positions of a whole market. The option may be regarded as a secondary construction. The sell and buy of “Call” in the option trade assumes the short position. On the other hand, the sell and buy of “Put” does the long position. However, this system cannot work well without any normal operation of the futures market. The crash of the tulip bubble in the Netherlands was a failure of the options market. To complement the lack of the mark to the market, computer risk management was added from the 1970s in the US.

5.2.1.3 Massive Sell Orders from Erroneous Human Operations

Before HFT became dominant, the market experienced several cases of erroneous human operations. There were two particularly famous cases in Japan.³

Dentsu in 2001: During the initial public offering of advertising giant Dentsu, in December 2001 a trader at UBS Warburg, the Swiss investment bank, sent an order to sell 610,000 shares in this company at ¥1 each, when he intended to sell one share at ¥610,000. The bank lost £71 million.

J-COM in 2005: During the initial public offering of J-COM, on December 8, 2005, an employee at Mizuho Securities Co., Ltd. mistakenly typed an order to sell 600,000 shares at ¥1, instead of an order to sell one share at ¥600,000. Mizuho failed to catch the error; the Tokyo Stock Exchange initially blocked attempts to cancel the order, resulting in a net loss of US $347 million shared between the exchange and Mizuho.⁴

At the aftermath of the J-COM case, both companies undertook to try to deal with their troubles: lack of error checking, safeguards, reliability, transparency, testing, and loss of both confidence and profit. On December 11, 2005, the Tokyo Stock Exchange acknowledged that its system was at fault in the Mizuho trade.⁵

Collapses may happen in spite of renovations. In a traditional auction, cancelation is essential for efficient transactions. In high-frequency trades, however,

cancelation will be limited to algorithm agents and given a special strategic role. As the market structure becomes more hierarchical, we must look again at the conventional definitions of trade tools. The rule of stop high and stop low will no longer exercise the same control of the range of price movements. In millisecond transactions, a traditional guideline cannot work effectively, and it will be difficult to prevent flash crashes. New trade tools must be developed to re-domain the market (Fig. 5.9).

The gigantic expansion of the market in size and complexity is closely connected with the evolution of algorithmic transactions and the speeding-up of both processing and order strategies, i.e., the creation of an artificial intelligence-dominant economy. This evolution has resulted in some hidden and unmeasurable destructive power in the invisible market, quite similar to large earthquakes. This is particularly observed as a rapid development of SBSs. In these situations, humans must co-evolve with artificial intelligence, otherwise they will be dominated by it.

5.2.1.4 Implementation of Relaxed Static Stability in the Financial Market

Shadow Computer control made possible a new generation of military aircraft designed to be inherently unstable, i.e., in technology jargon, to have relaxed static stability. This gave an advantage. Just as you can maneuver an unstable bicycle more easily than a stable tricycle, you can maneuver an inherently unstable aircraft more easily than a stable one. The new controls act to stabilize the aircraft much as a cyclist stabilizes a bicycle by offsetting its instabilities with countering motions. A human pilot could not react quickly enough to do this; under the earlier manual technology, an inherently unstable aircraft would be unflyable (Arthur 2009, p. 73).
5.3 Some Instances of Technological Innovations in the Complex Market Economy

Mainzer recommended considering fluctuations of a minimum quantity in a critical situation. Symmetry breaks down by itself. The minimum quantity then evolves by itself. This is crucial for the whole universe. It is reasonable to look to biological evolution to understand the evolution of technology. These views led to the idea of creative coincidence in human history. This idea may also be applied to technological innovation, when its application suggests that a new idea will replace an old through Schumpeter’s famous creative destruction.⁶

5.3.1 Redundancies and the Depth of Logic Contained in a Complex System

The redundancy of a DNA sequence is the result of largely random evolution over millions of years. The algorithmic complexity of DNA will therefore lie between random and perfect regularity of structure. Because of its redundancy, the algorithmic complexity of a DNA sequence $s$ is reduced, and can be described by a shorter program $s^*$, which is a more compact source of information that requires effort to implement the complete description of $s$. Genetic information is therefore an example of a sequence with a high logical depth (Mainzer 2007, p. 156).

There are two important ideas here: **high and low complexity** and **logical depth**.

**High/low complexity:** Define the probability $P_s$, by a program that produces a random sequence $s$ as output. By $l$ we denote the length of program delivered. For binary sequences of length $l$ there are $2^l$ possible combinations. Then $2^{-l}$ is the probability that the program $p$ of length $l$ is chosen, which is equally likely among the sequences of this length. $P_s$ can now be the sum of all probabilities $2^{-l}$, for all random programs of output length $l$. We can then prove that $P_s = 2^{-K(s)}$ in respect of the algorithmic complexity $K(s)$ of the sequence $s$. It follows that sequences with low complexity are more likely to occur than those with high complexity. We can then assume that in the sets of two sequences, the shorter sequence is the most likely to occur. The shorter programs contribute more to total $P_s$ over all programs with output $s$ (Mainzer 2007, p. 157).

**Logical Depth:** With the algorithmic probability $P_s$ for a randomly generated program’s output, we now have a measure of the logical depth of $s$. A sequence $s$ has logical depth when the largest proportion of $P_s$ is contributed by short programs that require a large number of computational steps to produce $s$. For example, DNA sequences that have evolved over millions of years, with

---

⁶See Schumpeter (1942, p. 82).
many redundancies and contingencies, can generate an entire organism through compact programs that require an enormous number of computational steps, and therefore have considerable logical depth.

5.3.2 Innovation and Techno-Culture

A complex system with a greater logical depth may add some elaboration at each stage. A degree of redundancy is essential for generating innovation, so that a series of small changes will often lead to much greater innovation. In other words, a greater logical depth in engineering may lead to more elaboration with high precision and accuracy. These elaborations in essence are indifferent to a pecuniary motive or any market mechanism. It is clear that this approach underlies the traditional Japanese techno-culture.

5.3.2.1 Economy in Modern Technology

Proposition 5.1. Technology is not a subclass of the economy; it is a superclass.

This idea was put forward by Brian Arthur in his book The nature of technology. He suggested that the economy arose from its technologies, including the productive methods and legal and organizational arrangements that we use to satisfy our needs. We can say that technology creates itself from itself. Early technologies used earlier primitive technologies as components. These new technologies in time become possible components - building blocks - for the construction of further new technologies. Some of these in turn go on to become possible building blocks for the creation of still newer technologies.

Over the last ten years or so, there has been a change in the nature of innovation. Before then, it generally took a long time to create a basic and usable piece of technology from a basic principle, so companies used their technology for as long as possible, improving its efficiency but not changing it radically. Now, however, it is much easier to turn a new basic principle into stable technology, but there are correspondingly greater numbers of possible new technologies, making selecting the right one much more difficult.

5.3.3 A Creative Coincidence Connected with Hayabusa’s Return and JAXA’s Evolution

On June 14, 2010, the Japan Aerospace Exploration Agency (JAXA) celebrated the return of Hayabusa, a planetary exploration spacecraft. It returned from the asteroid 25143 Itokawa, named in honor of Professor Hidao Itokawa (1912–1999).
5.3 Some Instances of Technological Innovations...

He worked as an aircraft engineer at the Nakajima Aircraft Company, and was responsible for the design of the Nakajima Ki-43 Hayabusa Oscar (Type 1 Fighter), the main fighter plane used by the Japanese Air Force throughout the Second World War. Between 1941 and 1967, he held a professorship at the department of engineering at the University of Tokyo. Despite a ban on research and education on aeronautics after the war, he continued his work.\footnote{See ISAS (2005). This is the special issue of honor of Dr. Itokawa.}

With the ban on the study of aeronautics, Itokawa first moved to the area of acoustic engineering and attempted to construct a violin. He then used the ban as an impetus to move into the study of rocketry. This gave rise to a Japanese project around innovation in aerospace, which grew into the Japan Aerospace Exploration Agency (JAXA). The success of the Hyabusa spacecraft started from the launch of his 23-cm pencil rocket on April 12, 1955 (Fig. 5.10).

The Hayabusa mission nearly collapsed several times. At each critical point, engineers cooperated to find a solution and successfully guide Hayabusa. One case provides an example of the use of creative coincidences (Fig. 5.11).
JAXA had several important objectives for both astrophysical observations and aeronautical engineering. Tasks included ion engine operation, autonomous and optical navigation, deep space communication, and close movement of objects with low gravity, as well as a challenging task of making contact with the surface of an asteroid, though this was regrettably not fully achieved.

The Hayabusa project was characterized by the following **four new innovative requirements**:

1. A round trip between planets using ion engine thrust.
2. Autonomous navigation and guidance from optical observations.
3. Sampling the planetary surface.
4. Re-entry into the atmosphere from an interplanetary trajectory and collection of the sampler.

These required equipment including the ion engine, sampler horn, target marker, and capsule (see Fig. 5.12).

Hayabusa had four ion engines, Thrusters A to D. Each thruster consisted of two units: ion source and neutralizer. Thruster A became inactive shortly after launch, and Thruster B’s neutralizer deteriorated in April 2007, rendering it inactive. Thruster C also had to be stopped for safety reasons. On the return trip on November 4, 2009, Thruster D stopped automatically when the neutralizer deteriorated. JAXA nearly had to abandon the spacecraft. Fortunately, a solution was found, and Hayabusa successfully re-entered Earth’s atmosphere, enabling recovery of samples.

The recovery was possible because of a small idea: combining the neutralizer unit from Thruster A with the ion source unit of Thruster B to create a new combined thruster, in a cross-operation. This massively increased the redundancy of operation.9

A cross-operation seems a smart solution, but could never be realized without valid circuit recombination. The basic technology required for this, a relay, was too heavy to include in the spacecraft, rendering a cross-operation impossible. However, the project had incorporated a new technology to achieve the same results, a single tiny diode embedded in the engine.10

This problem could not have been solved without this new technology. A cross-operation is a mathematical solution, but also required engineering expertise and forethought to make it possible. The use of mathematical solutions may therefore be useless without further engineering (Fig. 5.13).

---

8See Kuninaka (2009).
9See JAXA Press Releases (2009).
10See Kuninaka (2006).
5.3 Some Instances of Technological Innovations...

Fig. 5.12 Ion engine, sampler horn, target marker, and capsule. Source: ISAS/JAXA Space Exploration Center (2008)

5.3.4 An Assessment of the Hayabusa Mission

The history of the development of space technology in Japan was surely sparked by Itokawa’s creative inspiration as well as his interest. The Government was always reluctant to support it, so that it was largely driven by the passion of scientists and engineers. This particular mission relied on cooperation with private sector engineers. Put another way, this consortium was motivated by common academic interests.

Development was only able to continue thanks to the cost performance of solid fuel rockets. The idea of solid fuels for rocket propelling was developed by Itokawa, as a response to cost pressures after the war. JAXA’s Government-supplied budget is estimated as only one tenth of NASA’s budget. Its performance is therefore truly efficient.
5.3.4.1 **Vertical Integration and the Financial Circuit**

According to econophysicists, the market economy usually produces a power law distribution of price fluctuations, share holdings, capital size, and sales, which is characterized by a heavy or fat tail. In other words, there is strong vertical integration in the market network. This is valued in modern markets, even though it quickly leads to a monopolistic concentration. However, vertical integration in the public sector is often criticized.

**Hayek’s self-organization** is triggered by a creative coincidence in the market function, but prefers a special distribution of a heavy tail to a Gaussian distribution. Put another way, his self-organized market is often doomed to become a vertically integrated economy that is heavily biased towards the financial circuit. Paradoxically, the result may not be an efficient market. A modern financial circuit cannot guarantee stability of the market or system.
5.3 Some Instances of Technological Innovations...

Table 5.6 Barabási rule

<table>
<thead>
<tr>
<th>Step 0</th>
<th>There is a perfect graph with m nodes $Km$ in an initially random network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Add a new node to span a new link to one of the existing nodes with a probability $\pi(k_i) = \frac{k_i}{\sum_i k_i}$</td>
</tr>
<tr>
<td>Step 2</td>
<td>Iterate Step 1 until a specified number is reached</td>
</tr>
</tbody>
</table>

Source: Barabási and Albert (1999)

Table 5.7 Ohkubo rule

<table>
<thead>
<tr>
<th>Step 0</th>
<th>A random network of $N$ nodes and $M$ edges is initially given.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>An edge $l_{ij}$ is randomly specified.</td>
</tr>
<tr>
<td>Step 2</td>
<td>Replace the edge $l_{ij}$ with an edge $l_{im}$ whose node $m$ is chosen randomly with a probability.</td>
</tr>
</tbody>
</table>

Source: Ohkubo and Yasuda (2005) and Ohkubo et al. (2006)

Note: Here $k_m$ is the degree of node $m$, and $\beta_m$ a fitness parameter of node $m$

5.3.4.2 A Complex Network in Terms of a Polya Urn Process and a Non-growing Network

Complex network analysis cannot explain circumstances with deep logic and information. However, it can be used to show that a monetary exchange system can generate a feedback system.

Recent developments of network analysis since Barabási and Albert (1999) have tended to focus on how a network could generate a scale-free property. As complex network analysis showed, a huge network system is vulnerable to collapse following strategic aggression. This kind of vulnerability is closely connected to the scale-free property. We therefore examine a preferential attachment to a random network to see whether the network evolves as a scale-free system. Reciprocity is one of the important features of networking, and may be dominated by a type choice of preferential attachment. This analysis suggests management of complex networks is possible. More importantly, it is essentially equivalent to a Polya urn process, which may also generate the increasing returns or winner-takes-almost-all process of a modern production system.

A monetary network may be regarded as a random exchange system. Customers, however, often like a preferential attachment. Suppose that a network never grows but reconnects mutually. Ohkubo and Yasuda (2005) and Ohkubo et al. (2006) proposed a new rule for this non-growing network and also proved that the rule was equivalent to a Polya urn process. We presume a fitness parameter distribution $\phi(\beta) = \{\beta_i\}$ as time independent that is capable of creating a different possibility for network formation. The rule will be shown in Tables 5.6 and 5.7:
Table 5.8 A Polya urn rule

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Initially, there is a partition of $N$ urns (boxes) and $M$ balls</td>
</tr>
<tr>
<td>1</td>
<td>Randomly draw ball $n_{ij}$ from any box $i$</td>
</tr>
<tr>
<td>2</td>
<td>Replace it in a selected urn with a transition rate $W_{ni} \rightarrow W_{n_{i+1}} = (n_{ni} + 1)^{\beta}$</td>
</tr>
</tbody>
</table>

Source: Ohkubo and Yasuda (2005) and Ohkubo et al. (2006)

Table 5.9 Equivalence of the preferential random network and the Polya urn process

<table>
<thead>
<tr>
<th>Network reconnection</th>
<th>Polya urn process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 0</td>
<td>$N$ nodes and $M$ edges</td>
</tr>
<tr>
<td>Step 1</td>
<td>Randomly choose an edge $l_{ij}$</td>
</tr>
<tr>
<td>Step 2</td>
<td>$\Pi_m \propto (k_{m+1})^{\beta_m}$</td>
</tr>
</tbody>
</table>

5.3.4.3 The ‘Rich Get Richer’ Process

Now we can show the rich get richer phenomenon on $k$ under the Barabási rule. The average (mean) is written as:

$$<k> = 2mt.$$ (5.1)

The probability that the node added at the $s$-th step becomes degree $k$ is $p(k, s, t)$ and the probability that the node of degree $k$ increases its own degree is $\frac{k}{2t}$. We then have the equation:

$$p(k, s + 1) = \frac{k - 1}{2t} p(k - 1, s, t) + \left(1 - \frac{k}{2t}\right) p(k, s, t),$$ (5.2)

which gives the solution:

$$P(k) = \frac{2m(m + 1)}{k(k + 1)(k + 2)}.$$ (5.3)

If $k$ is a large number, it leads to the power law (Table 5.8):

$$P(k) \propto k^{-3}.$$ (5.4)

Table 5.9 shows the ways in which the preferential random network and Polya urn process are directly comparable or equivalent.
5.3.4.4 Interpretation by a Polya Urn Process

Suppose we have a system where there are $N$ urns and $M$ balls. We denote the number of balls in the urn $i$ by $n_i$. The total number of balls is:

$$M = \sum_{i=1}^{N} n_i.$$  \hfill (5.5)

We then define the energy of each urn as:

$$E(n_i) = -\ln(n!).$$  \hfill (5.6)

The Hamiltonian of the whole system is then:

$$H = \sum_{i=1}^{N} E(n_i).$$  \hfill (5.7)

The definition of energy may lead to attaching the un-normalized Boltzman weight to urn $i$:

$$p_{n_i} = \exp[-\beta_i E(n_i)] = (n_i!)^{\beta_i}.$$  \hfill (5.8)

The use of the heat-bath rule gives the transition rate $W_{n_i \rightarrow n_i+1}$ from the state $n_i$ to $n_i+1$ (Fig. 5.14).

5.4 Key Ideas for the New Economics

The Polya urn process provides us with potentially rich applications to analyze many important problems. In the above, we have examined some particular variations of the Polya urn process. However, one of major challenges to generalize our stochastic arguments depends on how to deal with the unknown agents. Without improving such an analytical modeling, we could not deal with how mutants will emerge. So our new economics need to learn how to formulate the unknown agents in the Polya urn process. This idea will generalize the framework of economic science.

5.4.1 The Economics of the Master Equation and Fluctuations

The stochastic evolution of the state vector can be described in terms of a master equation, such as the Chapman-Kolmogorov differential equation system. The master equation leads to aggregate dynamics, from which the Fokker-Planck
The equivalence between a network connection and a Polya urn process can be derived. We can therefore explicitly argue that there are fluctuations in a dynamic system. These settings can be connected with some key ideas, making it possible to classify agents by type in the system, and to track the variations in cluster size (Aoki and Yoshikawa 2006).

In Aoki’s new economics, there are exchangeable agents in a combinatorial stochastic process, like the urn process. The exchangeable agents emerge by the use of random partition vectors, as in statistical physics or population genetics. The partition vector provides us with information about the state. We can then argue the size-distribution of the components and their cluster dynamics with the exchangeable agents.

Suppose a maximum countable set in which the probability density of transition from state $i$ to state $j$ is given. In this setting, the dynamics of the heterogeneous interacting agents gives the field where one agent can become another. Unknown agents can also be incorporated.
5.4.1.1 A $K$-Dimensional Polya Distribution

We make a $K$-dimensional Polya distribution using parameter $\theta$. We then have a transition rate:

$$w(n, n - e_i + e_j) = \frac{n_i n_j + \theta_j}{n(n - 1 + \theta)}$$

Where:

$$n = n_1 + \cdots + n_K; \quad \theta_j > 0$$

And:

$$\theta = \sum_{j} \theta_j$$

The next step is a jump Markov process in a stationary state:

$$\pi(n) w(n, n - e_i + e_j) = \pi(n - e_i + e_j) w(n - e_i + e_j, n) \quad (5.9)$$

$$\pi(n) = \frac{w(n - e_i + e_j, n)}{w(n, n - e_i + e_j)} \pi(n - e_i + e_j) \quad (5.10)$$

We therefore have the stationary distribution:

$$\pi(n) = \frac{n!}{\theta^{[n]} n_1!} \prod_{i=1}^{K} \frac{\theta_i^{[n_i]}}{n_i!}$$

Where:

$$\theta^{[n]} = \theta(\theta + 1) \cdots (\theta + n - 1)$$

5.4.2 A General Urn Process

Suppose that balls (or agents) and boxes (or urns) are both indistinguishable. We then have a partition vector:

$$a = (a_1, a_2, \ldots, a_n)$$
$a_i$ is the number of boxes containing $i$ balls. The number of balls is:

$$\sum_{i=1}^{n} ia_i = n$$

The number of categories is:

$$\sum_{i=1}^{n} a_i = K_n.$$  

$K_n$ is the number of occupied boxes. The number of configurations is then:

$$N(a) = \frac{n!}{\prod_{j=1}^{n} (j!)^{a_j} a_j!} = \frac{n!}{(1!)^{a_1}(2!)^{a_2}\cdots(n!)^{a_n}a_1!a_2!\cdots a_n!}$$

### 5.4.2.1 A New Type Entry in an Urn Process

Let $a$ be a state vector. Suppose that one new type agent enters an empty box. We then have the equation:

$$w(a, a + e_1) = \frac{\theta}{n + \theta}$$  \hspace{1cm} (5.13)

Suppose too that an agent enters a cluster of size $j$. We add one to size $j + 1$ while reducing one from size $j$. We then have:

$$w(a, a + e_{j+1} - e_j) = \frac{ja_j}{n + \theta}$$  \hspace{1cm} (5.14)

Suppose that an agent leaves a cluster of size $j$. We add one to size $j - 1$ while reducing one from size $j$. We then have:

$$w(a, a - e_j + e_{j-1}) = \frac{ja_j}{n}$$  \hspace{1cm} (5.15)

We then use Ewens’ sampling formula:

$$\pi(a) = \frac{n!}{\theta^{[n]}} \prod_{j=1}^{n} \left( \frac{\theta}{j} \right)^{a_j} \frac{1}{a_j!}$$  \hspace{1cm} (5.16)
Where:

\[ \sum_{j=1}^{n} j a_j = n; \sum_{j=1}^{n} a_j = K_n \]

The probability that the number of clusters is \( k \)

The probability that the number of clusters is \( k \) will be the sum of a newcomer who comes in a new cluster with probability \( \frac{n}{n + \theta} \) and one who comes in an existing cluster with probability \( 1 - \frac{n}{n + \theta} \):

\[ q_{n,k} := \Pr(K_n = k | n) \] (5.17)

\[ q_{n+1,k} = \frac{n}{n + \theta} q_{n,k} + \frac{\theta}{n + \theta} q_{n,k-1} \] (5.18)

In this case, the boundary conditions will be:

\[ q_{n,1} = \frac{(n - 1)!}{\theta^{[n]}} \] (5.19)

\[ q_{n,n} = \frac{\theta^n}{\theta^{[n]}} \] (5.20)

The solution is then:

\[ q_{n,k} = \frac{\theta^k}{\theta^{[n]}} c(n, k) \] (5.21)

where

\[ c(n + 1, k) = n c(n, k) + c(n, k - 1) \]

\[ \theta^{[n+1]} = \sum_{m=0}^{n} s(n, m) \theta^{m} = \theta(\theta + 1)(\theta + 2) \cdots (\theta + n - 1)(\theta + n) \]

\[ = \theta^{[n]}(\theta + n) = \theta \cdot \theta^{[n]} + n \cdot \theta^{[n]} \]

\[ \theta \cdot \theta^{[n]} = \sum_{m=0}^{n} s(n, m) \theta \cdot \theta^{m} = \sum_{m=0}^{n} s(n, m) \theta^{m+1} \]

\[ = \sum_{m-1=0}^{n} s(n, m - 1) \theta^{m} = c(n, k - 1) \]
The final equation is called the signless Stirling Number of the first kind.

5.4.3 Pitman’s Chinese Restaurant Process

We suppose that there are an infinite number of round tables in the Chinese restaurant that are labeled by an integer from 1 to \( n \). The first customer, numbered 1, takes a seat at table number 1. Customers No. 1 to No. \( k \) in turn take their seats at their tables from No. 1 to No. \( k \). The \( c_j \) customers take their seats at the \( j \)-th table (Pitman 1995; Yamato and Shibuya 2000, 2003).

The next arriving customer has two options: either a seat at the \( k \)-th table, with the probability:

\[
\frac{\theta + k\alpha}{\theta + n}
\]

or at table \( j \), one of the remaining tables \( j = 1, \ldots, k \), with the probability:

\[
\frac{c_j - \alpha}{\theta + n}
\]

Two parameters, \( \theta \) and \( \alpha \), are used, so we obtain the solution:

\[
\frac{n!\theta^{[k:\alpha]}[n]}{\theta[n]} \prod_{j=1}^{n} \left( \frac{(1 - \alpha)[j-1]}{j!} \right)^{c_j} \frac{1}{c_j} \tag{5.22}
\]

Where:

\[
\theta^{[j]} = \theta(\theta + 1) \cdots (\theta + j - 1)
\]

\[
\theta^{[j:\alpha]} = \theta(\theta + \alpha) \cdots (\theta + (j - 1)\alpha)
\]

Ewens’ sampling formula Ewens (1972) gives the invariance of the random partition vectors under the properties of exchangeability and size-biased permutation. The Ewens sampling formula is the case with one parameter, a special case of two-parameter Poisson-Dirichlet distributions:

\[
\frac{n!}{\theta^{[n]}} \prod_{j=1}^{n} \left( \frac{\theta}{j} \right)^{a_j} \frac{1}{a_j!}
\]
In the case of the two-parameter Poisson-Dirichlet model, we will be faced with a non-averaging system as the limit.

Summing up, we should be keen to incorporate these ideas into the standard framework of economic science, as Aoki and Yoshikawa (2006, 2007) challenged.

References

r=0
ISAS/JAXA Space Exploration Center (ed.) (2008) The great challenges of ‘HAYABUSA'(world’s First asteroid sample return mission); Return of the Falcon (The story of ‘Hyabusa’, a spacecraft to explore an asteroid. The pamphlet to DVD


Chapter 6
The Complexities Generated by the Movement of the Market Economy

Abstract In previous chapters we have seen the advent of a new society, with the growing complexity of the market and technological innovations. We demonstrated the move away from the Gaussian world towards that of heavy tail distribution. To fully understand the latter distribution, we need to recognize a generalized central limit theorem, where the Gaussian distribution is simply a special case. We are therefore able to investigate several stable distributions and expand our idea of price equilibrium as a stable distribution. We will also discuss the central issue of how the market economy can generate complex dynamics. We use the trader dynamics system designed by Philip Maymin, and recreate his simulation dynamics for an automaton. This explicitly takes into account traders’ decisions in combined multiple layers: actions, mind states, memories of the trader and the market. These factors are normally regarded as influential components that govern price movements. In financial transactions, we dispense with psychological effects at individual and aggregate levels. These may no longer be secondary effects.

6.1 A Brief Summary of the Efficient Market Hypothesis

6.1.1 Disengagements from the Efficient Market Hypothesis

We noted in Chap. 5 that the efficient market hypothesis cannot really explain the current state of financial markets, nor the global financial crisis, because it relies on the idea of the market operating by a series of random walks.

A stock price time series under the random walk hypothesis may be represented as:

\[ P_t = P_{t-1} + \epsilon_t = P_0 + \epsilon_0 + \epsilon_1 + \cdots + \epsilon_t \]  

(6.1)

\( P_t \) will follow a Gaussian distribution. Here \( \epsilon \) is regarded as white noise. The expected price is calculated around \( P_0 \), so the expected distance from \( P_0 \) has a finite value. Each price change will be estimated from the next \( \epsilon_0 + \epsilon_1 + \cdots + \epsilon_t \). Each price change will therefore be the stochastic variant of a normal distribution. This is the simplest illustration of the efficient market hypothesis.
There are several variants of this:

Auto-regressive model $\text{AR}(p)$: $y_t = a_1 y_{t-1} + \cdots + a_p y_{t-p} + \xi_t$

Moving average model $\text{MA}(p)$: $y_t = \xi_t - b_1 \xi_{t-1} - \cdots - b_p \xi_{t-p}$

Auto-regressive moving average model $\text{ARMA}(p,q)$: $y_t = a_1 y_{t-1} + \cdots + a_p y_{t-p} + \xi_t - b_1 \xi_{t-1} - \cdots - b_p \xi_{t-p}$

These auto-regressive models are linear, and are therefore described as linear auto-regressive models. There are several differences between $\text{AR}$ and $\text{MR}$. The auto-correlation function will decrease and approach zero in $\text{AR}$, but in $\text{MR}$, it will instantly be zero whenever the parameters exceed a critical point. The partial auto-correlation function in $\text{AR}$ will instantly be zero whenever the parameters exceed a critical point, but will decrease and approach zero in $\text{MR}$. The power spectrum has a peak in $\text{AR}$, which will emerge around zero if the fluctuations are small. In $\text{MR}$, a valley emerges around zero if the fluctuations are larger (Fig. 6.1).

We now show a simulation of $\text{AR}(2)$, i.e., a second-order auto-regressive simulation. This Wolfram Demonstration uses a random variable drawn from a normal density with mean zero and variance unity. It is governed by the equation:

$$ AR(2): y_t = a_1 y_{t-1} + a_2 y_{t-2} + \xi_t \quad (i = 3, \ldots, N) \quad (6.2) $$

Here $\xi_t$ is a random variable, $N$ is the length of the series and the constants $a$ are auto-regressive. If $a_1 + a_2 < 1$, $a_2 - a_1 < 1$, and $-1 < a_2 < 1$, then the

---

1A more sophisticated version of this is the auto-regressive integral moving average model, $\text{ARIMA}(p, q, r)$. $\text{MA}(1)$ is an approximate function of $\text{AR}(\infty)$.
series is stationary. A series of length 400 is created in every case in this Wolfram Demonstration.

We then move to a more sophisticated regression, with two different regimes into and out of which the variable can move. Each regime follows a similar Gaussian/random process. We remove the trend item from the model, and make the process stationary and geometrically ergodic. In this case, however, the expected value $E(x)$ generated by the two-regime model is not necessarily zero. The regimes may then be described by the condition:

$$x_{t-1} < k$$  \hspace{1cm} (6.3)

$$x_{t-1} \geq k$$  \hspace{1cm} (6.4)

We therefore obtain a two-regime threshold auto-regressive (TAR) first-order process (Fig. 6.2):

$$x_t = \begin{cases} 
\alpha_1 + \beta_1 x_{t-1} + \epsilon_t < k \\
\alpha_2 + \beta_2 x_{t-1} + \epsilon_t \geq k
\end{cases}$$  \hspace{1cm} (6.5)

We set the constants $\alpha_i$ as 0, and $\beta_1 = -1.5$ in the first regime, $\beta_2 = -0.5$ in the second regime. The series contains large upward jumps when it becomes negative, and there are more positive than negative jumps.

We then denote the actual profit of security $i$ at term $t$ by $r_{it}$. It is then assumed that:

$$r_{it} = E(r_{it} \mid I_{t-1}) + \epsilon_{it}$$  \hspace{1cm} (6.6)

Here $I_t$ is the information set available everywhere at term $t$. $r_{it}$ is a random error at $t$. It then holds that:

**Efficient Market Hypothesis (Fama 1970).**

1. If the market were efficient, all usable information could affect market prices.

$$E(\epsilon_{i,t+s} \mid I_{t-1}) = 0, \hspace{0.5cm} s = 1, 2, \ldots$$  \hspace{1cm} (6.7)

$$E(\epsilon_{j,t+k} \epsilon_{i,t} \mid I_{t-1}) = 0, \hspace{0.5cm} k = 0, 1, 2, \ldots$$  \hspace{1cm} (6.8)

$$E(\epsilon_{i,t+k} \epsilon_{jt} \mid I_{t-1}) = 0, \hspace{0.5cm} k = 0, 1, 2, \ldots$$  \hspace{1cm} (6.9)

2. The efficiency of the market can measure all the responses of prices, given all the information $I_t$.

3. If anyone can obtain excess profit from some particular information, this market is not efficient in relation to this kind of information.

$$E(\epsilon_{it} E(r_{it} \mid I_{t-1}) = E(\epsilon_{it}) E(r_{it} \mid I_{t-1}) = 0$$  \hspace{1cm} (6.10)

\[2\] Also see Fama (1965).
A Weak Form. Using price information, technical analytical traders can earn excess profit.

A Semi-strong Form. Basic information gives any trader a chance to earn excess profit.

A Strong Form. Insider information gives any trader a chance to earn excess profit.
6.2 Moving Away from the Social Philosophy Around the Gaussian Distribution

Fig. 6.3 Self-similarity in the random walk: scale = 5, 50, 500. http://demonstrations.wolfram.com/SelfSimilarityInRandomWalk/

In summary, in the world where the hypothesis holds, errors should not be systematically generated. This implies that:

\[ E(\epsilon_t \mid I_{t-1}) = 0 \]  \hspace{1cm} (6.11)

6.1.1.1 Self-similarity in the Random Walk

A random walk process never excludes the emergence of self-similarity. We may therefore find a strange attraction in the efficient market hypothesis (Fig. 6.3). It is clear that:

Random walk ⇒ Fractal motion
Fractal motion \(\neq\) Random walk

We now show a simulation of a random walk to generate self-similarity. This is unchanged for any scaling as well as period length.

In the next section we verify the historical implications of the Gaussian distribution and the central limit theorem (CLT), and will then argue a generalization of the CLT. We will see that any distribution other than the Gaussian may benefit from a generalized CLT. It is also clear that the CLT would be valid even if there was no variance.

6.2 Moving Away from the Social Philosophy Around the Gaussian Distribution

6.2.1 The Historical Penetration of the Gaussian Distribution and Galton’s Ideas

Lambert Adolphe Jacques Quételet (1796–1874) is still well known as the proposer of the Quételet index or Body Mass Index. His enthusiasm penetrates various social ideas even to this day. However, advances in science are uncovering the fictitious background to Quételet’s ideas.
A key role is played by the astronomer and sociologist Lambert Adolf Jacob Quételet, who, in 1835, for the first time, challenged others to think of the ‘average person’ as well as statistics. Does it matter that there are large error calculations in the natural sciences or in investigations of ‘normal distributions’ in society? It is today hardly comprehensible that natural and social scientists of the nineteenth century took large numbers and normal distributions as a universal rule in nature and society (Mainzer 2007, p. 48; trans. by the author).

Modern probability theory suggests that the Gaussian or normal distribution is not universal, instead taking the normal distribution as a special case stable distribution. This also changed the implications of the CLT. Unfortunately, many economists still hold to Galton’s conviction of the dominance of the normal distribution by means of the CLT.\(^3\)

---

\(^3\)The British physician and natural scientist Francis Galton (1822–1911), who first tried to measure intelligence, stressed the law of the large number: “I hardly know anything other than this whose imagination is influenced in such a way like the marvelous form of cosmic order, which is expressed by the law of the error frequency. It prevails with bright serenity, and complete self-denial in the middle of the wildest confusion. The greater the rabble, the greater the obvious anarchy. Its influence is surely the highest law of senselessness”.
6.2 Moving Away from the Social Philosophy Around the Gaussian Distribution

Galton demonstrated an approximation of a binomial distribution using a nail board (see Fig. 6.4). Each time a ball hits a nail, it has a probability of bouncing either left or right. The balls accumulate in the bins at the bottom.

A visualization of the Bernoulli distribution can be traced back to Galton. On a perpendicularly standing board, nails are set at a constant distance in \( n \) parallel rows (see Fig. 6.4). Each nail is exactly in the gap between two nails in the row above. If balls of the diameter of the distance between nails are dropped from a funnel above the board, they may fall through without being affected by friction from the nails. If a ball hits a nail, it can fall either to the right or to the left. This procedure repeats itself in each row. The ball therefore falls in a random way on the nail board, being diverted to the right or left with equal probability. After going through the \( n \) rows, the balls land in the \( n + 1 \)-th box. A ball would fall into box \( i \) if it were diverted \( i \) times to the right and \((n - i)\) times to the left. This process leads to a Bernoulli distribution of the balls (Mainzer 2007, p. 49; trans. by the author).

It is interesting to use simulation to verify and compare the frictionless and frictional cases. Even in the physical world, using an experimental method, we cannot reproduce an ideal state, and so it is hard to justify a world of normal distributions (Fig. 6.5).

Galton’s belief therefore does not hold, as the CLT states. The CLT can be generalized to random variables without second (and possibly first) order moments and the accompanying self-similarity of the stable family. Even if the first moment is infinite, we can find an existing stable distribution, such as the power law distribution. A new definition of stable distributions thus contains the normal distribution as a special case. These findings were largely responsible for the focus on Gaussian distributions. The role of non-Gaussian distributions was first noted

---

**Fig. 6.5** A Galton board with frictionless and frictional cases. The details of parameters in this simulation are: In the left panel, earth acceleration 2; friction coefficient 0; initial position 0; initial velocity 0; initial angle \( \pi/2 \); In the right panel, earth acceleration 2; friction coefficient 2; initial position 0; initial velocity 0.5; initial angle \( \pi/2 \); Note: Nail parameters for both figures: thickness 0.2; strength 0.1; surface softness 2. [http://demonstrations.wolfram.com/GaltonBoard/](http://demonstrations.wolfram.com/GaltonBoard/)
by Benoît Mandelbrot, who proposed that cotton prices followed an **alpha-stable distribution** with $\alpha$ equal to 1.7, a Lévy distribution.\(^4\)

### 6.2.1.1 Generalized Central Limit Theorem (GCLT)

It has often been argued recently that the "tail" behavior of sums of random variables determine the domain of attraction for a distribution. The classical CLT is merely the case of finite variance, where the tails lie in the domain of attraction of a normal distribution. Now, however, interest has moved to infinite variance models, bringing a new generalized CLT. This describes distributions lying in the domain of attraction of a stable distribution, or:

A sum of independent random variables from the same distribution, when properly centered and scaled, belongs to the domain of attraction of a stable distribution. Further, the only distributions that arise as limits from suitably scaled and centered sums of random variables are stable distributions.\(^5\)

We now consider the generalization of the CLT. This explanation draws heavily on John D. Cook’s note “Central Limit Theorems”.\(^6\)

The propagation of the idea of a Gaussian distribution was inseparable from the powerful role of the CLT. More interestingly, however, it has recently transpired that the theorem was applicable to non-identically distributed random variables, meaning that distributions other than Gaussian are possible in the context of the CLT. Under the generalized CLT, distributions other than Gaussian will be guaranteed a similar powerful role.

The classical CLT requires variables with three conditions: independence, identical distribution and finite variance. Let $X_n$ be a sequence of independent, identically distributed (IID) random variables. It is assumed that $X$ has finite mean $\mu = E(x)$ and finite variance $\text{var}(X) = \sigma^2$. We then observe its normalized average as:

$$Z_n = \frac{X_1 + X_2 + \cdots + X_n - n\mu}{\sigma \sqrt{n}}$$  \hspace{1cm} (6.12)

The CLT then states that $Z_n$ converges point-wise to the cumulative distribution function (CDF) of a standard normal (Gaussian) random variable.\(^7\)

The CLT is not necessarily limited to identically distributed random variables with finite variance. There are, therefore, two different types of generalized CLT, for

---

\(^4\)See Mandelbrot (1963). Lévy distributions are also found in nature, e.g., in spectroscopy, with many cases seen in solar flare data. See Lévy (1925).

\(^5\)http://demonstrations.wolfram.com/GeneralizedCentralLimitTheorem/.

\(^6\)See Cook (2010).

\(^7\)See Appendix for “An elementary derivation of the one-dimensional central limit theorem from the random walk”.

6.2 Moving Away from the Social Philosophy Around the Gaussian Distribution

non-identically distributed random variables and for random variables with infinite variance.

We now discuss a generalized theorem for non-identically distributed random variables. We assume that $X_n$ is a sequence of independent random variables, at least one of which has a non-degenerate distribution. We then define the partial sum and its variance:

\[
\begin{align*}
\text{the partial sum } S_n &= X_1 + X_2 + \cdots + X_n \\
\text{its finite variance } s_n^2 &= \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2
\end{align*}
\]

We use the **Lindeberg-Feller theorem** to check the convergence of $S_n/s_n$ using the Lindeberg condition for the CLT for non-identically distributed random variables. The Lindeberg condition is both necessary and sufficient for convergence. Let $F_N$ be the CDF of $X_n$, i.e., $F_n(x) = P(X_n < x)$. The Lindeberg condition is:

\[
\lim_{n \to \infty} \frac{\sigma_n^2}{s_n^2} \int_{x > \epsilon s_j} x^2 dF_j(x) = 0 \text{ for all } \epsilon > 0. \tag{6.15}
\]

The Lindeberg-Feller theorem therefore holds and:

**Sufficiency.** If the Lindeberg condition holds, $S_n/s_n$ converges to the distribution of a standard normal random variable.

**Necessity.** If $S_n/s_n$ converges to the distribution of a standard normal random variable, then the Lindeberg condition holds.

We now remove the assumption of finite variance while keeping that of independence, i.e., $X_0$, $X_1$, $X_2$ following IID:

**Definition 6.1.** The distribution of random variables is called **stable** if there is a positive $c$ and a real $d$ such that $cX_0 + d$ has the same distribution as $aX_1 + bX_2$.

In stable distributions, the shape is characterized by four parameters. The exponent parameter $\alpha$ is probably best known. The parameter will work in the range of $0 \leq \alpha \leq 2$. If $\alpha = 2$, it follows a Gaussian distribution. For $\alpha < 2$, then:

The Probability Density Function (PDF) for $\alpha < 2$ is asymptotically proportional to $|x|^{-\alpha-1}$.

The CDF for $\alpha < 2$ is asymptotically proportional to $|x|^{-\alpha}$ as $x \to \pm \infty$.

All stable distributions other than the normal will therefore have thick tails and non-finite variance. In general, density functions for stable distributions are not possible in closed form, although there are three exceptions: normal, Cauchy, and Lévy distributions.

We can check the convergence of the aggregation of random variables $X_i$ for $\alpha < 2$. Let $F(x)$ be the CDF for $X$, and the slowly varying function $h(x)$ so that:

\[
\frac{h(cx)}{h(x)} \to 1, \text{ as } x \to \infty \text{ for all } c. \tag{6.16}
\]
The necessary and sufficient conditions on \( F(x) \) may therefore be:

\[
F(x) = (c_1 + o(1))|x|^{-\alpha}h(x) \text{ as } x \to -\infty \tag{6.17}
\]

\[
1 - F(x) = (c_2 + o(1))x^{-\alpha}h(x) \text{ as } x \to \infty \tag{6.18}
\]

\( F(x) \) will therefore look something like \( |x|^\alpha \) in both the left and right tails. The aggregate random variables will be similar to the limiting distribution, giving a generalized CLT for infinite variance.

Figure 6.6 shows a simulation of the generalized CLT on the Pareto distribution:

The power law differs from the power distribution. The probability density for values in a power distribution is proportional to \( x^{a-1} \) for \( 0 < x \leq 1/k \) and zero otherwise.

---

\(^8\)\( o(1) \) represents a function tending to 0.

\(^9\)http://mathworld.wolfram.com/ParetoDistribution.html.

\(^{10}\)See Nolan (2005, 2009).

\(^{11}\)The power law differs from the power distribution. The probability density for values in a power distribution is proportional to \( x^{a-1} \) for \( 0 < x \leq 1/k \) and zero otherwise.
This will give rise to some technical difficulties, because of the characteristic functions for stable distributions. A stable distribution is defined in terms of its characteristic function $\phi(t)$, which satisfies a functional equation where for any $a$ and $b$ there are $c$ and $h$ such that $\phi(at)\phi(bt) = \phi(ct)\exp(iht)$. The general solution to the functional equation has four parameters. Stable distributions allow $0 < \alpha \leq 2, -1 \leq \beta \leq 1$, where $\mu$ is any real number, and $\sigma$ any positive real number. A characteristic function of stable distributions is a continuous $\alpha$:

$$
\exp \left( i\mu t - \sigma |t| \left( 1 + 2\frac{\beta}{\pi} \text{sgn}(t) \log(|t\sigma|) \right) \right) \text{ for } \alpha = 1
$$

$$
\exp \left( i\mu t - |t\sigma|^\alpha \left( 1 + i\beta \tan \frac{\pi \alpha}{2} \text{sgn}(t)(|t\sigma|^{1-\alpha} - 1) \right) \right) \text{ for } \alpha \neq 1
$$

This can be written in closed form in terms of the four parameters mentioned above. In general, however, the density functions for stable distributions cannot be written in closed form. There are three exceptions: normal, Cauchy, and Lévy distributions (Fig. 6.7).

The probability density for value $x$ in a Lévy distribution is proportional to:

$$
e^{-\sigma/(2(x-\mu))} \frac{1}{(x - \mu)^{3/2}} \quad (6.20)
$$

Here the Lévy distribution allows $\mu$ to be any real number and $\sigma$ to be any positive real number. A Lévy distribution with $\mu = 0, \sigma = 0.5$ is a special case of the inverse gamma distribution with $\alpha = 0.5, \beta = 0.5\sigma$.

---

12 A linear combination of independent identically distributed stable random variables is also stable.
13 As already noted, the generalized CLT still holds.
6.3.1 Hazard Rates

We focus on the lifetime of a person over the random variable $x$ with probability density function $f(x)$. The probability of surviving at least up to time $x$ is:

$$R(x) = \int_x^\infty f(x) \, dx = 1 - F(x) \quad (6.21)$$

$dx$ is a small interval of time, so the probability of death in time interval $dx$ is $f(x)$, the probability density function. Thus the instantaneous death rate at age $x$, i.e., the force of mortality, is defined as:

$$r(x) = \frac{f(x)}{1 - F(x)}. \quad (6.22)$$

If this ratio $r(x)$ is applied to the failure rate in manufacturing processes, it is called the hazard rate. It is clear that the distribution is possible whether the ratio rises or decreases with $x$; that is, whether it has increasing failure rate (IFR) or decreasing failure rate (DFR).\(^{14}\) The lognormal distribution becomes a DFR only beyond a point, while the Pareto distribution is a DFR throughout its range.

The probability density for value $x$ in a Pareto distribution is proportional to $x^{-\alpha}$ for $x > k$, and zero for $x < k$. The survival function for value $x$ in a Pareto distribution corresponds to:

$$\text{Pareto distribution} = \left(\frac{x}{k}\right)^{-\alpha} \quad \text{for } x \geq k$$

$$\text{Pareto distribution} = \left(1 + \frac{x - \mu}{k}\right)^{-\alpha} \quad \text{for } x \geq \mu$$

$$\text{Pareto Distribution} = \left(1 + \left(\frac{x - \mu}{k}\right)^{\gamma}\right)^{-\alpha} \quad \text{for } x \geq k \quad (6.23)$$

In the first form of the Pareto distribution $[k, \alpha]$, the mean is:

$$\frac{k\alpha}{1 + \alpha} \quad \text{for } \alpha \geq 1 \quad (6.24)$$

and the variance is:

$$\frac{k^2\alpha}{(-2 + \alpha)(1 + \alpha)^2} \quad \text{for } \alpha \geq 2 \quad (6.25)$$

\(^{14}\)DFR indicates that the ability to make more money might increase with one’s income (Singh and Maddala 2008, p. 28).
We now derive the Pareto distribution following Singh and Maddala (2008). It is convenient to transform $x$ into $z = \log x$. We then find the hazard rate with respect to log $x$ (Fig. 6.8):

$$ r^*(z) = \frac{dF}{dz} \frac{1}{1 - F} \quad (6.26) $$

Taking the negative form of the Pareto transformation, let $y, z$ be:

$$ y = -\log(1 - F) $$
$$ z = \log x $$
$$ y = f(z); y' > 0, y'' > 0 $$
By the log transformation of $x$, $y'$ is interpreted as the proportional failure. If we then suppose:

$$y'' = ay'(\alpha - y') \text{ i.e.,}$$

$$
\frac{y''}{y'} = a\alpha - ay' \tag{6.27}
$$

$a$ is constant. The formula then implies that the rate of change of $y'$ (the acceleration rate) depends on itself, and acceleration should move in a negative direction (Fig. 6.9).

To solve this differential equation, we make three assumptions about $r^*(z)$:

Assumption 1: $r^*(z)$ approaches a constant value asymptotically.
6.3 Heavy Tail Distributions with Heavier Randomness

Assumption 2: \( r^* (z) \) increases with accelerating rate, then with decreasing rate.

Assumption 3: \( \frac{r^*(z)}{dz} \) tends to zero as \( r^* (z) \to 0 \).

Taking account of \( ay'' = y''/(\alpha - y') \), Eq. (6.27) may be rearranged into:

\[
\frac{y''}{y'} + \frac{y''}{\alpha - y'} = a\alpha
\]

(6.29)

The integration of both sides will give:

\[
\log y' - \log(\alpha - y') = a\alpha z + c_1
\]

(6.30)

c_1 is an integral constant. The solution \( y' \) or the proportional failure is:

\[
Y' = \frac{\alpha e^{a\alpha z + c_1}}{1 + e^{a\alpha z + c_1}}
\]

(6.31)

We can then easily derive:

\[
\log y = \frac{1}{a} \log(1 + e^{a\alpha z + c_1}) + c_2
\]

(6.32)

c_2 is another integral constant.

We have then arranged to derive the distribution function. We note that:

\[
y = -\log(1 - F)
\]

\[
z = \log x
\]

We also note that:

\[
c = \frac{c_2 - c_1}{\alpha}
\]

\[
b = \frac{1}{e^{c_1}}
\]

The distribution function \( F \) is therefore described\(^{15}\):

\[
F = 1 - \frac{b^{1/a}}{(b + x^{a\alpha})^{1/a}}
\]

(6.33)

\(^{15}\)c is eliminated. If \( x = 0, F(0) = 0 \). It follows that \( c = b^{1/a} \).
6.3.2 An Alternative Derivation in View of Memoryless Processes

The Pareto process depends only on remaining one-sided, so \( \frac{dF}{dx} \) depends on \( 1 - F \) only. Singh and Maddala (2008, p. 31) called such a process memoryless:

\[
\frac{dF}{dx} = a(1 - F)^{1+1/a}. \tag{6.34}
\]

The most typical memoryless process may be the Poisson process:

\[
\frac{dF}{dx} = a(1 - F). \tag{6.35}
\]

The Weibull process is one into which memory is introduced:

\[
\frac{dF}{dx} = ax^b(1 - F). \tag{6.36}
\]

The motion of \( x \) affects its behavior. In general, in an integrated view of memoryless and memory processes, a more general formulation expression is:

\[
\frac{dF}{dx} = ax^b(1 - F)^c. \tag{6.37}
\]

Specifically, the values of \( a, b, c \) will be verified by a concrete form of the process generating a distribution.

Now we allocate further:

\[
a_1 = \frac{1}{b}, a_2 = a\alpha, a_3 = \frac{1}{a}
\]

It then follows that:

\[
\frac{dF}{dx} = \frac{a_1 a_2 x^{a_2 - 1}}{(1 + a_1 x^{a_2})^2} \tag{6.38}
\]

Here \( a_1 = \left( \frac{\alpha}{\lambda_0} \right)^{\alpha} \) and \( a_2 = \alpha. \)
The distribution Eq. (6.38) is then rewritten into:\(^{16,17}\):

\[
F = 1 - \frac{1}{(1 + a_1 x^{a_2})^{a_3}}
\] (6.39)

### 6.3.3 Some Empirical Findings in the Market

Empirical findings on the Pareto–Lévy distribution were first confirmed by Mandelbrot (1963), who examined (a) the New York Cotton Exchange, 1900–1905, (b) an index of daily closing prices of cotton on various exchanges in the US, 1944–1958 (published by Hendrik S. Houthakker), and (c) the closing price on the 15th of each month at the New York Cotton Exchange, 1880–1940 (published by the US Department of Agriculture). The unit of time \(t\) in the time series is different for each. Figure 6.10 shows all three cases (a)–(c) with positive and negative axes.

Notice that the variables representing the change rates in terms of logarithm are taken as differently as follows:

\(\begin{align*}
(a) & \quad X = \log Z(t + 1) - \log Z(t) \text{ is daily unit based.} \\
(b) & \quad X = \log Z(t + 1) - \log Z(t) \text{ is daily unit based.} \\
(c) & \quad X = \log Z(t + 1) - \log Z(t) \text{ is monthly unit based.}
\end{align*}\) (6.40)

It may also be helpful to reproduce the scheme of random process classes given by Mantegna (2000), Swell (2011) (Fig. 6.11).

The PDF of returns for the Shanghai market data with \(\triangle t = 1\) is compared with a stable symmetric Lévy distribution using the value \(\alpha = 1.44\) determined from the slope in a log–log plot of the central peak of the PDF as a function of the time increment. The agreement is very good over the main central portion, with deviations for large \(z\). Two attempts to fit a Gaussian are also shown. The wider Gaussian is chosen to have the same standard deviation as the empirical data. However, the peak in the data is much narrower and higher than this Gaussian, and the tails are fatter. The narrower Gaussian is chosen to fit the central portion, but the standard deviation is now too small. It can be seen that the data have tails that are much fatter and have a non-Gaussian functional dependence (Johnson et al. 2003; Swell 2011) (Figs. 6.12, 6.13, 6.14, and 6.15).

---

\(^{16}\)It is clear that \(F \rightarrow 1\) as \(x \rightarrow \infty\).

\(^{17}\)See Champernowne (1953) and Fisk (1961).
6.4 Alternative Interpretation: Trader Dynamics to Generate Financial Complexity

This section is inspired by Philip Maymin’s ingenious demonstrations in the Wolfram Demonstration Project. He attempted to formulate a single autonomous engine, based on the application of automata, to recreate complex stock price fluctuations. This helps us to understand the essence of the financial market mechanism by taking account of volatile shifts in traders’ minds, together with memory effects. The mind state may often accompany the practical employment of a typical technical analytical agent.

Maymin has focused on the actual aspect of traders’ decisions in terms of the combining of multiple layers: actions, mind states, and memories of traders and the market. The environment, here the market, is generated by a rule, but not by factors such as drift and other random factors that often appear in statistical time series analysis. If the market is rising, there are two possible kinds of action. One is a trend-follower action: to buy rising stocks. This may reflect a bearish or pessimistic mind. The other is the contrarian action to sell on the way up, or to buy on the way down. This may reflect a bullish or optimistic mind.
The resulting behavior of the trader depends on the mind state, whether bearish or bullish. Whatever the mind state, behavior may also be influenced by the past (memories) and the market hysteresis. We call this the internal rule. Maymin then gives a rule of shift of mind between several states, which plays a role as a transducer in the automaton. On the first shift downwards following an upward movement, for instance, the mind may move from optimistic to pessimistic. The trader will be induced to change his attitude from contrarian to trend-follower. Similarly, the same player, faced with an upward movement after a series of downward movements, will be induced to move from trend-follower to contrarian. This flow could cause the market to shift back to the original state. This suggests the shift rule will decide whether the market changes direction. In the model, such a floating rule is sensitive to the market change. It is evident that the emerging or floating shift rules will arrange the market in a periodic way. More importantly, the model incorporates these induced behaviors. The interactions between mind state, market situation, the trader’s memory and the market history can therefore be included.\(^\text{18}\) The trader will follow an individual pattern linked to past history.

\(^{18}\)The number of states may be more than two.
Fig. 6.12  Shanghai index. Cited from Fig. 4 of Swell (2011)

Fig. 6.13  All data from the Dow Jones Index. Source: Wolfram Alpha. http://www.wolframalpha.com/input/?i=dow+jones

Figure 6.16 shows this idea in more detail.
6.4 Alternative Interpretation: Trader Dynamics to Generate Financial

Fig. 6.14 Last year’s distributions of the Dow Jones Index on Dec 14, 2013. Source: Wolfram Alpha http://www.wolframalpha.com/input/?i=dow+jones

6.4.1 Rules to Be Specified

Even if we assume the simplest dealing, we would require several behavioral rules to achieve buy, sell and hold, taking into account the market state as well as a trader’s memory. The trader’s current thinking could be modified during dealing. We usually set several specifications for the trader’s dealing rules:

1. Action rules depending on the mind state:
   a. Internal rule
   b. Shift rule (transducer)

2. Look-back rule: the trader’s memory of action associated with the hysteresis of the market

   In the case of two actions {buy, sell}, two states {bearish, bullish}, and {three-period memory}, there are 256 possible rules, including overlapping ones. Some
rules may be more important than others. We usually have two rules for state of mind: one that gives the internal rule, and one the shift rule between the different states of mind. In reality, however, there are many combinations of mind state and market state, as well as actions of buy and sell.

The investor is modeled using an iterated finite automaton with only two states: an optimistic contrarian state and a pessimistic trend-following state. The trader starts each day in the optimistic contrarian state, which sells on upturns and buys on downturns, and remains in this state as long as the market keeps going up.

We take first a simple example. Maymin (2007d: Caption comment) described this model as:

Pick the rule he follows and the initial conditions for the first three days of the week. The trader looks back over the past three days in considering what to do. The big circles are states, like the emotions of the trader. He always starts in state 1 and then follows the arrows that correspond to the market movement (UP or DOWN) of the day under consideration. The yellow state is the one he was in when he started thinking about the day in question,
and the one with the square in it is the one he will move to, based on the yellow arrow that he follows. His “current thinking” is shown on the yellow arrow, too (BUY or SELL), but he only makes his final decision when he has reached the last day that he considers.

- **Action rule**
  
  Internal rule of the trader mind: In the bearish state, the trader sells when the market is UP. In the bullish state, the trader sells when the market is DOWN.
  
  Shift rule of the trader mind: Shift to the bearish state so that the trader buys when the market is UP. Shift to the bullish state so that the trader buys when the market is DOWN.

- **Look-back rule**
  
  The last three units of time are used for the look-back.

According to Maymin, we can imagine the initial conditions shown in Fig. 6.17.
"The trader looks at what happened on Wednesday..."

The investor follows these transitions by looking at the past few days of price changes, starting with the most recent day. The decision to buy or sell is modeled as his current thinking as he goes through the past few days, and once he reaches the last day of his look-back window, he makes his final decision to buy or sell. The next day, he starts all over.\footnote{Note in the Snapshots how sometimes volatility starts small and then increases, and sometimes it starts large and decreases. A price series that appears stable can suddenly collapse, and after a collapse can remain low, or can come back up. Because the investor looks back a fixed number of days, the price series is guaranteed to eventually cycle. However, this cycle length can be quite long. For example, a 15-business-day look-back, absent any external changes, will cycle in about 130 years. (Maymin 2007a: Details)}
In this instance of, the transition of the market will be shown in terms of the *transducer* finite automata:

\[
\{state_1, input\} \rightarrow \{state_2, output\}
\]  

(6.41)

Here, the trader is pessimistic and remains in this state as long as the market keeps going down.

In the left-hand side of Fig. 6.17, “[o]n the first downtick, it transitions to the other state, the pessimistic trend-following state. This state is trend-following because it buys on upticks and sells on downticks”. In the right-hand side, “[o]n the first uptick, it transitions back to the original state” (Maymin 2007a: Details). The model can therefore generate periodic motions.

In this rule:

- state 1 is characterized as an **up-absorbing** state.
- state 2 is characterized as a **down-absorbing** state.

Consequently, the rule could give the following reduced rule:

1. If the preceding two events are the same, i.e., both up or both down, the trader will sell.
2. If the preceding two events are different, i.e., one up and one down, the trader will buy.
6.4.1.1 An Alternative Interpretation Based on Tick Data

If each event is replaced by a tick,\(^{20}\) Maymin suggests:

An investor observes two consecutive ticks. If they are the same sign, i.e., either both up
or both down, then he sells. Otherwise he buys. However, his order does not take effect for
\(w - 1\) ticks. Put another way, his orders face a delay of \(w - 1\) ticks, so the minimal model
formalizes an investor who looks at consecutive ticks to decide his position and faces a short
delay either in mental processing or order execution (Maymin 2011a, p. 10).

\[
\{1, 1\} \rightarrow \{1, 0\}, \{1, 0\} \rightarrow \{2, 1\}, \{2, 1\} \rightarrow \{1, 1\}, \{2, 0\} \rightarrow \{2, 0\}. \tag{6.42}
\]

Here the first component is the state, 1 (bearish) or 2 (bullish). The second is the
trader’s action keeping up with the market state:

1: buy at up
0: sell at down

The trader will follow the **shift rule** as shown by \(\Rightarrow\) (see Fig. 6.18).

We now extend the look-back windows to 10 days or ticks, obtaining a new time
series (see Fig. 6.19).

6.4.1.2 A Generalization of the Simplest Trader Dynamics

Maymin proposed to define the domain of his speculative financial market as:

An investor with \(s\) internal states and \(k\) possible “actions” with base \(b\) and having a look-
back window of \(w\) days is modeled as an **iterated finite automaton (IFA)**.

---

\(^{20}\)Ticks are approximately of the order of seconds.
Employing the Wolfram IFA numbering scheme, the IFA is run as:

```math
ToFARule[m_Integer, {s_Integer, k_Integer}] :=
Flatten[MapIndexed[{1, -1} #2 + {0, k} ->
Mod[Quotient[#1, {k, 1}], {s, k}] + {1, 0} &,
Partition[IntegerDigits[m, s k, s k], k], {2}]]
```

$s$ represents the state of mind of the trader, which is sometimes bearish, sometimes bullish. $k$ represents the action of dealing, usually “buy”, “sell”, or “hold”, although it may be accompanied by the “limit order” or the “market order”. For simplicity, we assume dealing is to the market order only, because this is often genuinely possible. When $k = 2$ (Maymin 2011a), the possible actions are buy and sell. Maymin also proved that the possible actions are buy, sell, and hold if $k = 3$. He introduced a scale of strength of buy or sell, where the possible actions are buy, buy more, sell, and sell more, if $k = 4$.

In general, if $k$ is even, the possible actions are $k/2$ types of buys and $k/2$ types of sells, with differing strengths. If $k$ is odd, the possible actions are the same as for $k - 1$ with the additional possibility of doing nothing, i.e., hold.

There is a basic property of these new trader dynamics, that “[the trader] never learns his own rule”. In this modeling, however, the trader is based on “technical analysis”, and will trade in reference to a self-chosen technical indicator (Maymin 2011a, p. 4). In Maymin’s simple model, the trader will not change rule until there is a big loss. It follows that:

From his point of view, he put in an order to buy and was simply unlucky enough not to have been filled before the market moved away. This happens to him every day, whether he wants to buy or sell (Maymin 2011a, p. 4).

### 6.4.1.3 How Do the New Traders’ Dynamics Differ from Previous Models?

The new dynamics proposed by Maymin decisively differ from previous ideas as they contain rules. In many game theoretic formulations, the players are allowed to change moves by checking the ongoing development of the price. In this case, we specify that the trader can decide his market order, i.e., whether he can buy or sell, by looking back at the preceding price series. In other words, his decision depends on his previous memory. Given the preceding time series DOWN, UP, UP, the trader

---

21The strength of a buy or sell is an exponent of the base $b$. For example, when $b = 3$ and $k = 4$, the four possible actions are $-3, -1, 1,$ and $3$, meaning sell three times more than normal, sell normally, buy normally, or buy three times more than normal. If $k = 6$, then two additional actions are $-9$ and $9$. According to Maymin, “[i]f a normal buy would increase the market price by one percent, a nine times more fervent order would increase the market price by nine percent” (Maymin 2011a, p. 4).
will buy because of an expectation that the price will go up further. However, his memory is limited to three elements of the last price movement (Table 6.1).

The consequence may not differ from the new results, but the choice of the shift rules for mind transition could give rise to a big change in the price series (Fig. 6.20).

### 6.4.1.4 Maymin’s Justification for the Simplest Trader Dynamics

In the financial market, as described by Maymin, there is no need for multiple traders to create complex price fluctuations. We assume only the initial price ahead of the opening price quotation, on which the trader can act. The preceding price series motivates the trader to make further orders. Maymin (2007a,b,c,d; 2011a,b) gave this price information in the up and down form: \{up, down\}. Even this price series is created by the trader himself. The simplest model uses only a single trader, but this may be replaced with the point that in the preceding session, some trades

### Table 6.1 Transitory evolution of memory in traditional trader dynamics

<table>
<thead>
<tr>
<th>Rounds</th>
<th>BUY</th>
<th>BUY</th>
<th>SELL</th>
<th>BUY</th>
<th>BUY</th>
<th>SELL</th>
<th>BUY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D D U</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>D U U</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>U U U</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>U U D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>U D U</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>D U U</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>U U U</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Underlined letters indicate the current event on Up or Down.

**Fig. 6.20** A time series generated
were made, whether our particular trader was involved or not. After moving to the opening price, only one trader will continue to give further orders. In the futures market, self-dealing is acceptable, and in a double auction, it is possible to have simultaneous moves of buy and sell, usually called the day trader method. It is therefore possible to assume a sole agent in the market. Here the interactive effects caused by different agents may not be explicit but the interaction between buy and sell definitely emerges. We can therefore employ the single market simulation to investigate how and why a complex price fluctuation can be created. The market dealings allow moves of only sell, buy or hold, although there are also two kinds of limit order and the market order, as shown in Chap. 4. A trader may sometimes secure his profit by executing market orders. We therefore take an example of a single trader with market orders. This setting may be reasonable if we comply faithfully with the basic properties of the financial market.

This market will be reflected by the emotions of the trader, which essentially coincides with Lux (2009) and Hawkins (2011), where the market could be characterized by the simulation method of emotional interaction irrelevant to the real economy. This is an essential property of the financial market. In the event, through this kind of experiment, we can recognize the new proposition:

**Proposition 6.1.** The market is not capable of creating price fluctuations. Instead, the situation is the other way round. A trader, even if there is only one, can create the market, which is simply an institutional arrangement.

It is important to understand that the market in itself is not capable of creating anything. The agent does not have to be human. In high-frequency trading (HFT), the main agents are algorithmic ones. This proposition does not imply that human behavior can create a market fluctuation.

Another important feature of the financial market could be compatible with the **sock puppet** idea.

**Proposition 6.2.** A monologue, i.e., a single trader dynamic, can move the market, whether the action is judged illegal or not.

A sock puppet means “an online identity used for purposes of deception”. We assume that a sock puppet is the single trader in our market, and call the single trade dynamics required to generate a complex stock price fluctuation “sock puppet dynamics”.

---

22 In the Tokyo Stock Exchange, as set out in Chap. 4, the morning or afternoon session under the double auction rule will start after a batch auction on the preceding 30 min of orders.

23 In HFT, as set out in Chap. 4, a cross-operation is feasible if it can happen more quickly than a trading time unit.

24 According to Wikipedia, “The term - a reference to the manipulation of a simple hand puppet made from a sock - originally referred to a false identity assumed by a member of an internet community who spoke to, or about, himself while pretending to be another person”.
In dealings, the trader must have a strategy. The standard classical strategies are still applied in the market, although they date back to the era of the classical stock exchange. They may be used not only by human agents but also by machine or algorithm agents, and as we saw in Chap. 4, the random strategy or zero-intelligent strategy can work well.25

The trader can be given alternative options from the same past history, including towards the trend or against it, either the trend strategy or the anti-trend strategy. Either way, the decision could be canceled out before the simulation in either the classical auction or HFT. Throughout, the trader is never required to hold to the first choice of strategy, but can switch strategy either emotionally or intelligently. Needless to say, in the market, an intelligent strategy will never guarantee profits, even if the strategy is very sophisticated. Random strategies, on average, minimize the total loss because of naïve risk management.

6.4.2 **Complexities in a Dealing Model of an Iterated Finite Automaton**

First we need to define complexity. With a look-back window of \( w \) with \( k \) possible market movements each day, there are only \( k^w \) distinct possible price histories on which the trader bases a decision. After \( k^w \) days of history, the time series of price changes must have cycled. A complex series is one that takes a long time to cycle, and a simple series cycles quickly.

For example, a rule that always buys will cycle in a single day. Even though the price continues to rise to a unique level each day, the changes are constant. Here, using Maymin’s simulator,26 we can verify some findings of complex price motions, a periodic motion (see Fig. 6.21).

---

25 The U-Mart experiment is helpful in understanding how various random behaviors could intermediate a coordination of contracts in dealings.

26 The minimal model of simulating prices of financial securities using an iterated finite automaton, where length of generated series = initial condition.
6.4.2.1 Redundancies and the Depth of Logic Contained in a Complex System

This modeling has definitely generated complex price fluctuations. In general, however, it may be that there are actually not so many long complex price motions. Such an observation is usually true of other chaotic movements. Sequences with low complexity are more likely than those with high complexity, as we saw with DNA in a previous chapter (Mainzer 2007, p. 156).

**Redundancies and the depth of logic contained in a complex system** Redundancy mediated by a coincidence may bring something new. A combinatorial rearrangement found among redundancies suggests the potential for a new idea. If each program to generate a given consequence is smaller, the number of steps calculated by these programs will be increased. As a generated system becomes more complicated, the procedure to reveal the desired system will have a deeper logic, so a simpler program can create a complex system. This is why an organism or a gene can construct a huge complex constitution with a considerably deeper logic in spite of its simple procedure.

Recently, complex systems have often been analyzed as complex networks, but the present form of network analysis is insufficient because of the depth of logic and information. Network analysis gives useful findings, but does not yet have a tool to represent the depth of logic and information (Figs. 6.22, 6.23, and 6.24).

These events may be easily compared with those generated by Turing Machine (Turing 1936–37, 1937). A Turing machine “consists of a line of cells known as the ‘tape’ with an active element called the ‘head’ that can move back and forth and can change the colors of the tape according to a set of rules. Its evolution can be represented by causal networks that show how the events update” (Zeleny 2005). Among 4096 rule, Zeleny produced various causal networks of the next kinds: (1) repetitive behavior, (2) a binary counter, (3) a transient, (4) a stable structure from a random initial condition, and (5) arrows jumps backwards and forwards several levels. For reference, we reproduce the case (5) of three states with two colors (Fig. 6.25):

Here the change of mind state of the trader dynamics may correspond to the head change of Turing machine. Thus it evolution will be similar to the evolution of mind state in the trader dynamics. Thus we may expect that our trader dynamics can make a causal complex network. It may be also natural to argue NP (Non-deterministic Polynomial) complexity in non-deterministic Turing machine (Maymin 2011b).

6.4.2.2 Definition of Complexity in Simple Trader Dynamics

**Definition 6.2 (Maymin’s Complexity).** Complexity in a $k$-action generated price series for a look-back window $w$ means that the periodicity of the rule is greater than $kw/2$. A rule with the maximal complexity would have a periodicity equal to the maximal periodicity (cycle length) of $kw$, i.e., $2w$. Usually, cycle length $\leq$ the maximum $2w$. Generation of periodicity depends on the initial condition.
Definition 6.3 (A Relaxed Definition). The minimal complex model must have at least two states and two actions.

Lemma 6.1 (Complexity in the Price Series). Complexity in the price series requires $s \geq 2$ and $k \geq 2$. Cycling does not mean that the market is modeled as repeating, only that this is a characteristic of the model. We can always choose sufficiently large $k$ and $w$ such that the cycle length is longer than the history of the market under consideration.\textsuperscript{27}

\textsuperscript{27}k = 15 and $w = 15$ means the maximal cycle length is longer than the age of the universe in seconds.
6.4.2.3 A Rapid Growth of Complexity Due to HFT in Terms of Periodicity

We found a twofold period system in HFT. Inner look-back windows accompany each time unit, with a new window denoted by \( w \). We estimate the complexity by: \( kw\omega/2 > kw/2 \). In our simple trader dynamics with HFT, the complexity will therefore rapidly increase.
References

Lévy P (1925) Calcul des probabilités. Gauthier-Villars, Paris
Maymin PZ (2011b) Markets are efficient if and only if P=NP. Algorithmic Finance 1(1):1–11
Appendix A
Avatamsaka Stochastic Process*

A.1 Interactions in Traditional Game Theory and Their Problems

Even in traditional game theory, noncooperative agents usually encounter various kinds of interactions in generating a set of complicated behaviors. In the course of finding curious results, experimental economists are often involved in giving game theorists new clues for solving the games. Their concerns are limited to informational structures outside the essential game structure, e.g., using an auxiliary apparatus such as “cheap talks”, i.e., negotiations without costs. Traditional game theory is allowed to argue actual interactions extensively, but may run into difficulties, because the treatment of the information is merely intuitive and not systemic.

We exemplify one limitation in the case of a Nash equilibrium. If we are faced with multiple Nash equilibria, the concept of correlated equilibria may be activated. As Kono (2008, 2009) explored, however, this concept really requires the restrictive assumption that all players’ mixed strategies are assumed to be stochastically independent. Without this, a selection of equilibria may be inconsistent.¹ Plentiful realistic devices for strengthening traditional game theory cannot necessarily guarantee the assumption of stochastic independence of mixed strategies. So traditional theory may often be linked to evolutionary theory to argue realistic interactions.

A general framework for encompassing various contexts/stages can be systematically proposed by focusing on the concept of information partition, often used in statistical physics or population dynamics. It is easy to find such an application in the field of socio- and/or econo-physics. We can then argue that a player changes into another as the situation alters.

---

¹See Kono (2008, 2009).

*Appendix A first appeared in Aruka and Akiyama (2009).
The exchangeable agents change with the situation/stage.

The cluster dynamics change with the exchangeable agents.

The exchangeable agents emerge from the use of a random partition vector in statistical physics or population genetics. The partition vector provides us with information about the state. We can therefore argue the size-distribution of the components and their cluster dynamics with the exchangeable agents. We can link the cluster dynamics with the exchangeable agents. We then define a maximum countable set, in which the probability density of transitions from state $i$ to state $j$ is given. In this setting, dynamics of the heterogeneous interacting agents give the field where an agent can become another. This way of thinking easily incorporates the unknown agents, as Fig. 1.9 shows (Aoki and Yoshikawa 2006).

### A.1.1 A Two-Person Game of Heterogeneous Interaction: Avatamsaka Game

Aruka (2001) applied a famous tale from Mahayana Buddhism Sutra called *Avatamsaka*. Now I would like to illustrate the Avatamsaka game. Suppose that two men sat down face to face at a table, tied up except for one arm, then each was given an over-long spoon. They cannot serve themselves, because of the length of the spoon. There is food enough for both on the table. If they cooperate to feed each other, they can both be happy. This defines Paradise. Alternatively, one may provide the other with food but the second might not reciprocate, which the first would hate, leading to Hell. The gain structure will not depend on the altruistic willingness to cooperate.\(^2\) The payoff matrix of the traditional form will be (Tables A.1 and A.2):

The properties of the two games may be demonstrated by the relationships between $R, S, T, P$: Here we call

\[ D_r = P - S \]  

\(^2\)Recently, this tale has been cited in *Chicken Soup* (Canfield and Hansen 2001), a best-selling book in the United States.
the “Risk Aversion Dilemma” and

\[ D_g = T - R \]  \hspace{1cm} (A.2)  

the “Risk Seeking Dilemma”.

**Selfishness** cannot be defined without interactions between agents. The direction of the strategy will depend on the situation of the community as a whole. One agent’s selfishness depends on the other cooperating. A gain from defection can never be assured independently. The sanction for defection, as a reaction of the rival agent, never implies selfishness of the rival.\(^3\)

### A.1.2 Dilemmas Geometrically Depicted: Tanimoto’s Diagram

Tanimoto’s (2007) geometrics for the two-person game neatly describe the game’s geometrical structure. Given the payoff of Table A.1, his geometrics define the next equation system:

\[ P = 1 - 0.5r_1 \cos \left( \frac{\pi}{4} \right) \]  \hspace{1cm} (A.3)  
\[ R = 1 + 0.5r_1 \cos \left( \frac{\pi}{4} \right) \]  \hspace{1cm} (A.4)  
\[ S = 1 + rr_1 \cos \left( \frac{\pi}{4} + \theta \right) \]  \hspace{1cm} (A.5)  
\[ T = 1 + rr_1 \sin \left( \frac{\pi}{4} + \theta \right) \]  \hspace{1cm} (A.6)  

Here \( r = \frac{r_2}{r_1} ; r_1 = PS, r_2 = SM \).

Thus it is easily verified that spillovers of the Avatamsaka game are positive:

\[ \text{Spillover} = R - S = T - P > 0 \]

Each player’s situation can be improved by the other player’s strategy switching from \( D \) to \( C \), whether he/she employs \( D \) or \( C \) (see Aruka 2001, p. 118) (Fig. A.1).

---

\(^3\)The same payoff was used as the “mutual fate control” game by the psychologists Thibaut and Kelley (1959), revived by Mitropoulos (2004). However, the expected value of the gain for any agent could be reinforced if the average rate of cooperation were improved, in a macroscopically weak control mechanism, which cannot always guarantee a mutual fate. Aruka and Akiyama (2009) introduced different spillovers or payoff matrices, so that each agent may then be faced with a different payoff matrix.
A.1.2.1 The Path-Dependent Property of Polya’s Urn Process

The original viewpoint focuses on an emerging/evolving environment, i.e., path dependency. We focus on two kinds of averaging:

Self-averaging: Eventually, players’ behavior could be independent from others’.
Non-self-averaging: The invariance of the random partition vectors under the properties of exchangeability and size-biased permutation does not hold.

In the original stage, consider an urn containing a white ball and a red ball only. Draw out one ball, and return it with another of the same color to the urn. Repeat over and over again. The number of balls increases by one each time, so after the completion of two draws,

The total number of balls after the second trial = 2 + 1 + 1 = 4
The total number of balls after the n-th trial = n + 2

After the completion of n trials, what is the probability that the urn contains just one white ball? This must be equivalent to the probability that we can have n successive draws of red balls. It is therefore easy to prove that

\[ P(n, k) = \frac{1}{n + 1} \]  \hspace{1cm} (A.7)

This result shows that any event \((n, k)\) at the \(n\)-th trial can emerge, i.e., any number of white balls can go everywhere at the ratio of \(\frac{1}{n+1}\).

\[ \frac{1}{2} + 1 \frac{1}{2 + 1} \frac{1}{2 + 1 + 1} \cdots \frac{n}{n + 1} = \frac{1}{n + 1} \]  \hspace{1cm} (A.8)
Comparing this process with the market, it is clear that in the stock or commodity exchanges, for instance, any sequence of trades must be settled at the end of the session. Any trade, once started, must have an end within a definite period, even in the futures market. The environment in the market must be reset each time. In the market game, a distribution of types of trader can affect the trading results, but not vice versa. On the other hand, the Avatamsaka game in a repeated form must change its own environment each round. A distribution of types of agents can affect the results of the game, and vice versa. Agents must inherit their previous results. This situation describes the path dependency of a repeated game (Fig. A.2).

A.2 Avatamsaka Stochastic Process Under a Given Payoff Matrix

We apply the Polya urn stochastic process to our Avatamsaka game experiment to predict an asymptotic structure of the game. We impose the next assumptions for a while Aruka (2011, Part III).

Assumption 1 There are a finite number of players, who join the game as pairs.

Assumption 2 The ratio of cooperation, or C-ratio for each player, is in proportion to the total possible gains for each player.

As Aoki and Yoshikawa (2006) dealt with a product innovation and a process innovation, they criticized Lucas’s representative method and the idea that players face micro-shocks drawn from the same unchanged probability distribution. In light of their findings, I show the same argument in the Avatamsaka game with different payoffs. In this setting, innovations occurring in urns may be regarded as increases in the number of cooperators in urns whose payoffs are different. Moving on from a classical Polya urn process with a given payoff matrix (Aruka 2011, Part III), we then need the following formulae:

**Ewens sampling formula (Ewens 1972)** A $K$-dimensional Polya urn process with multiple payoff matrices and new agents allowed to arrive.
**Self-averaging** Eventually, however, players’ behavior can be independent of others’.

**Pitman’s sampling formula** Two-parameter Poisson-Dirichlet distribution.

**Non-self-averaging** The invariance of the random partition vectors under the properties of exchangeability and size-biased permutation does not hold.

According to Aoki and Yoshikawa (2007, p. 6), the economic meaning of non-self-averaging is important because such models are sample-dependent, and some degree of impreciseness or dispersion remains about the time trajectories even when the number of economic agents reaches infinity. This implies that focus on the mean path behavior of macroeconomic variables is not justified, which, in turn, means that sophisticated optimization exercises that provide us with information on means have little value. We call an urn state **self-averaging** if the number of balls in each urn could eventually be convergent.

**Definition A.1.** Non-self-averaging means that a size-dependent (i.e., extensive in physics) random variable $X$ of the model has a coefficient of variation that does not converge to zero as the model size tends to infinity. The coefficient of variation of an extensive random variable, defined by

$$C.V.(X) = \frac{\text{variance}(X)}{\text{mean}(X)},$$

is normally expected to converge to zero as model size (e.g., the number of economic agents) tends to infinity. In this case, the model is said to be **self-averaging**.

### A.2.1 Avatamsaka Stochastic Process Under Various Payoff Matrices

Next we introduce various payoff matrices into our Avatamsaka process. Let various types of different sized payoff matrices be $1, \cdots, K$. We then have similar gain-ratios of players with different payoff types for each number of agent with gain $i$. Every player who encounters an opponent must draw lots from any lottery urn. A couple of players can draw a lot arbitrarily, and then return them to the urn. It then holds that the greater spillover does not change Avatamsaka characteristics. However, different spillovers may change the players’ inclinations and reactions as shown in Fig. A.3. It is noted that we can get a PD game by extending the PR line to the north-east.

Suppose there are a number of different urns with various different payoff matrices, each of which has its own spillover size. A different spillover in our Avatamsaka game may change the inclinations of the players, but these inclinations are not necessarily symmetrical. An urn with a greater spillover might sometimes be more attractive for a highly cooperative player, because the players could earn greater gains in size. A less cooperative player might prefer to enter an urn with
a smaller spillover in a much higher level of cooperation, but he can change his mind and defect. Players may therefore have various plans. Any player depends on the state in which he remains or enters, while an urn, i.e., a payoff matrix, occurs stochastically.

By the time the $i$-th cooperation occurs, the total of $K_n$ payoff urns are formed in the whole game space, where the $i$-th payoff urn has experienced cooperation for $i = 1, 2, \ldots, K_n$. If we replace “innovations” with “increases in cooperation”, then by definition the following equality holds:

$$n_1 + n_2 + \cdots + n_k = n$$

(A.10)

when $K_n = k$. If the $n$-th cooperation creates a new payoff matrix (urn), then $n_k = 1$, and

$$n = \sum_j j a_j (n)$$

(A.11)

So there are a finite number of urns into which various types of payoff matrices are embedded for $1, 2, \ldots, K_n$.

In this new environment, we can have $n$ inventions to increase cooperation. In other words, the amount of cooperation $x_i$ in urn $i$ may grow, because of stochastic multiple inventions occurring within it.

**Assumption 3** (Aruka and Akiyama 2009, p. 157; Aruka 2011, p. 257) Cooper-ation accelerates cooperation, i.e., the greater the cooperation, the larger the total gain in the urn will be.

Because of an Avatamsaka property, we can also impose another assumption.
Assumption 4 (Aruka and Akiyama 2009, p. 157; Aruka 2011, p. 257) A player can compare situations between the urns by normalizing his own gain.

Under the new assumptions, we can prove the non-self-averaging property in our Avatamsaka game as

\[ X_n K_n + \beta \sum_{j}^n j a_j (n) \]  

(A.12)

Here \( \beta = \log(\gamma) > 0 \). It turns out that \( X_n \) depends on how cooperation occurs. Finally, according to Aoki and Yoshikawa (2007), the next proposition also follows:

**Proposition A.1.** In the two-parameter Poisson-Dirichlet model, the aggregate cooperation behavior \( X_n \) is non-self-averaging.
Appendix B
The JAVA Program of URandom Strategy

In order to be familiar with implementing a new strategy in the U-Mart, we need to construct a java program for a particular strategy. Here we merely demonstrate the URandom strategy. This strategy premises the next institutional properties as its default setting:

ReferenceBrand=Futures
TradingBrand=Futures
TradingType=NORMAL

#Price
Price.Buy.Average=Ask
Price.Buy.StandardDeviation=2.5Tick
Price.Sell.Average=Bid
Price.Sell.StandardDeviation=2.5Tick

#Volume
Volume.Buy.Max=10
Volume.Buy.Min=1
Volume.Sell.Max=10
Volume.Sell.Min=1

#Frequency
OrderFrequency=Fixed

(continued)

4See src/startegyv4/URandomStrategy.java in the U-Mart ver.4 system.
5See src/startegyv4/URandomStrategyProperty.txt in the U-Mart ver.4 system.
It is trivial that the institutional setting may be easy to be changed by assigning a different value in the file. As for “#Frequency”, it may change the frequency framing by either removing # or changing the given numerical values.

```java
package strategyV4;

import java.io.IOException;
import server.UBrandInformationManager;
import server.UMachineAgentInformation;
import server.UMartTime;

public class URandomStrategy extends UStandardAgent {
    private static final double CASH_LIMIT = 1 / 3;
    private int fCountOfOrderFrequency;
    private int fOrderFrequency;

    public URandomStrategy(int dummyParam) {
        super(dummyParam);
    }

    @Override
    public void setParameters(String parameters) throws IOException {
        super.setParameters(parameters);
        fCountOfOrderFrequency = 0;
        fOrderFrequency = getOrderfrequency();
    }
}
```

(continued)
@Override
public void action(UMartTime time,
UMachineAgentInformation machineAgentInfo,
UBrandInformationManager brandInfoManager) {
    super.action(time, machineAgentInfo,
                 brandInfoManager);

    if(isOrder()){
        //If the random value is zero, buy; if 1, sell.
        if(getRandomInteger(2) == 0){
            buyOrder(brandInfoManager);
        }
        else{
            sellOrder(brandInfoManager);
        }
    }

    /**
     * Check the timing whether order should be given or not.
     * @return
     */
    private boolean isOrder() {
        if (fOrderFrequency == fCountOfOrderFrequency) {
            fCountOfOrderFrequency = 0;
            fOrderFrequency = getOrderfrequency();
            return true;
        }
        fCountOfOrderFrequency++;
        return false;
    }

    /**
     * Decide the frequency of giving orders.
     * If the frequency is fixed, orders are given by a constant frequency.
     * If it is randomly generated, orders are given by the frequency generated by a normal random number.
     * @return
     */
    (continued)
private int getOrderfrequency() {
    int count;

    if (fSystemParameters.getProperty("OrderFrequency").equals("Fixed")){
        count = fSystemParameters.getIntProperty("OrderFrequency.Frequency");
    } else{
        count = (int) getRandomGaussian(fSystemParameters.getDoubleProperty("OrderFrequency.Average"),
                                        fSystemParameters.getDoubleProperty("OrderFrequency.StandardDeviation"));
    }

    if (count < 0) {
        return 0;
    } else{
        return count;
    }
}
Appendix C
An Elementary Derivation of the One-Dimensional Central Limit Theorem from the Random Walk

A particle starting at 0 jumps by moving randomly towards just +1 or −1 on the coordinate (−∞, +∞). Let \( k \) be the number of jumps in the positive direction, and \( l \) the number of jumps in the negative direction. We then denote these by \( N \) and \( m \):

\[
N = k + l, m = k - l
\]  

(C.1)

It then holds that \( m \) is odd if \( N \) is odd. The probability that a particle jumps \( n \) in the positive direction is set as \( P(n) \). The probability in the opposite direction is set as \( 1 - P \). The generated path will accompany its probability \( P^k \times (1 - P)^l \). There are also combinatorial pairs of different jumps from \( k = 0 \) to \( k = N/2 \):

\[
\binom{N}{k} = \frac{N!}{k!(N-k)!}
\]  

(C.2)

The probability that the particle jumped \( k \) times for +1 each, and \( l \) times for −1 each, is set as:

\[
\Pi_N(k) = \binom{N}{k} P^k (1 - P)^l
\]  

(C.3)

Taking into account that \( m = 2k - N \), it holds that:

\[
k = \frac{N + m}{2}, l = \frac{N - m}{2}
\]  

(C.4)

We have now calculated the probability of net jumps. This is verified by noting that (Fig. C.1 and Table C.1):

\[
m = 2k - N
\]  

(C.5)
Table C.1 The probability distribution of the one-dimensional random walk

<table>
<thead>
<tr>
<th>Steps</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/4</td>
<td>0</td>
<td>1/4</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1/8</td>
<td>3/8</td>
<td>0</td>
<td>1/8</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1/16</td>
<td>3/16</td>
<td>6/16</td>
<td>1/16</td>
<td>1/16</td>
<td>1/16</td>
</tr>
<tr>
<td></td>
<td>1/32</td>
<td>5/32</td>
<td>10/32</td>
<td>5/32</td>
<td>1/32</td>
<td>1/32</td>
</tr>
</tbody>
</table>

\[ \Pi_N(m) = \left( \frac{N}{\sqrt{2\pi m}} \right) P^{\frac{N+m}{2}} (1 - P)^{-\frac{N-m}{2}} \]  

For a rudimentary confirmation, we give the next formula. Let \( p_i \) be the probability that event \( i \) occurs. Taking a normalization rule \( \sum_{i=1}^{n} p_i(x_i) = 1 \), we reach the weighted arithmetic mean of \( x = (x_1, \cdots, x_n) \):

\[ E(x) = \frac{\sum_{i=1}^{n} p_i(x_i) x_i}{\sum_{i=1}^{n} p_i(x_i)} = \sum_{i=1}^{n} p_i(x_i) x_i \]  

The deviation from this mean value is defined as:

\[ \Delta x_i = x_i - E(x) \]
It then follows that:

\[ E(\Delta x_i) = \sum_i p_i(x_i) - E(x) = 0 \] (C.9)

The second moment is then defined as:

\[ E(\Delta x_i^2) = \sum_k (p_i(x_i) - E(x))^2 \] (C.10)

\[ = E(x^2) - E(x)^2 \] (C.11)

C.1 The Derivation of the Density Function of the Normal Distribution

In the following, we will derive the density function of the normal distribution. We give a Taylor expansion around the maximum \( k = \nu \) on the function \( \ln \Pi(k) \), for convenience of calculation, instead of \( \Pi(k) \).

\[
\ln \Pi(k) = \ln \Pi(\nu) + \frac{d \ln \Pi(\nu)}{dk}(k - \nu) + \frac{d^2 \ln \Pi(\nu)}{dk^2}(k - \nu)^2 + \cdots \quad (C.12)
\]

We replace \( \frac{1}{k!} \frac{d^k \ln \Pi(\nu)}{dk^k} (k - \nu)^k \) with \( b_k \), and \( k - \nu \) with \( \lambda \):

\[
\ln \Pi(k) = \ln \Pi(\nu) + b_1 \lambda + \frac{b_2}{2} \lambda^2 + \frac{b_3}{6} \lambda^3 + \cdots \] (C.13)

As the maximum is attained at \( k = \nu \), the necessary condition of the first derivative and the sufficient one of the second are fulfilled as:

\[ b_1 = \ln \Pi(\nu)' = 0; \quad b_2 = \ln \Pi(\nu)'' < 0 \] (C.14)

---

6 The next proof is inspired by the idea of Prof. Yoshihiro Nakajima, Osaka City University, who believes the necessity of a rudimentary understanding of the CLT by some self-contained proof (Nakajima 2011).
$b_2$ becomes negative, and hence may be rewritten as:

$$\frac{b_2}{2} \lambda^2 = -\frac{|b_2|}{2} \lambda^2$$  \hspace{1cm} (C.15)

We then assume that $\lambda = k - \nu$ is small enough to ignore $\lambda^3, \ldots, \lambda^k$, and we may then reach the next approximation:

$$\ln \Pi(k) \approx \ln \Pi(\nu) - \frac{|b_2|}{2} \lambda^2, \ i.e.,$$

$$\Pi(k) \approx \Pi(\nu)e^{-\frac{|b_2|}{2} \lambda^2}$$  \hspace{1cm} (C.17)

Finally, we transform $\sum_k^N \Pi(k)$ into a continuous integral form:

$$\sum_k^N \Pi(k) = \int_{-\infty}^\infty \Pi(k - \nu)d\lambda$$  \hspace{1cm} (C.18)

We note that:

$$\lambda = k - \nu, \ \sum_k^N \Pi(k) = 1$$  \hspace{1cm} (C.19)

It then by the famous integral formula on a Gaussian function holds that:

$$\sum_k^N \Pi(k) = \int_{-\infty}^\infty \Pi(\nu)e^{-\frac{|b_2|}{2} \lambda^2}d\lambda$$  \hspace{1cm} (C.20)

$$= \Pi(\nu) \sqrt{\frac{2\pi}{|b_2|}} = 1$$  \hspace{1cm} (C.21)

And then:

$$\Pi(\nu) = \sqrt{\frac{|b_2|}{2\pi}}$$  \hspace{1cm} (C.22)

We examine each item $\Pi(k)$.

$$\Pi(k) = \frac{N!}{k!(N-k)!} P^k(1 - P)^{N-k}$$
Use the next approximation: 
\[ \frac{\ln(k+1)!-\ln k!}{(k+1)-k} \]. As \( k \) becomes large enough, 
\[ \frac{d \ln k}{dk} \approx \ln k. \]

\[ \frac{d \ln \Pi(k)}{dk} = -\ln k - \ln(N - k) + \ln P - \ln(1 - P) \]

\[ \frac{d^2 \ln \Pi(k)}{dk^2} = -\frac{1}{k} - \frac{1}{N - k} \]

(C.23)

If \( \frac{d \ln \Pi(k)}{dk} \) = 0 at \( k = v \), it then follows that:

\[ v = PN \]

(C.24)

Then, at \( k = v \):

\[ b_2 = -\frac{1}{NP(1 - P)} \]

(C.25)

We then have:

\[ E(k^2) = \sum \Pi(k)k^2 \]

(C.26)

\[ = \sum_k \binom{N}{k} P^k (1 - P)^k k^2 \]

(C.27)

Taking into account the expression:

\[ P \frac{d}{dP} P^k = P_k P^{k-1} = k P^k, \left( P \frac{d}{dP} \right)^2 P^k = k^2 P^k \]

(C.28)

It follows:

\[ E(k^2) = \left( P \frac{d}{dP} \right)^2 \sum_k \binom{N}{k} P^k (1 - P)^k k^2 \]

(C.29)

\[ = \left( P \frac{d}{dP} \right)^2 (P + (1 - P))^N \]

(C.30)

\[ = \left( P \frac{d}{dP} \right) (PN(P + (1 - P))^{N-1}) \]

(C.31)

\[ = (PN)^2 + NP(1 - P) \]

(C.32)

It then holds:

\[ E(k^2) = v^2 + NP(1 - P) \]

(C.33)
Since:

\[ E(\Delta k^2) = E(k^2) - E(k)^2 \]  \hspace{1cm} (C.34)

It then holds:

\[ E(\Delta k^2) = E(k^2) - E(k)^2 \]  \hspace{1cm} (C.35)
\[ = \nu^2 + NP(1 - P) - \nu^2 \]  \hspace{1cm} (C.36)
\[ = NP(1 - P) \]  \hspace{1cm} (C.37)

We denote \( E(\Delta k^2) \) by \( \sigma^2 \). This creates the next formula for the normal distribution:

\[ \Pi(k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-\nu)^2}{2\sigma^2}} \]  \hspace{1cm} (C.38)

This is the density function of the normal distribution.

\[ \Pi(\nu) = \sqrt{\frac{|b_2|}{2\pi}} \]
\[ b_2 = \frac{1}{NP(1 - P)} \]
\[ \sigma^2 = E((\Delta k)^2) = 4NP(1 - P) \]

If \( p = 0.5 \) in the last equation, it holds that:

\[ \sigma^2 = E((\Delta k)^2) = N, \text{i.e.,} \]
\[ \sigma = E(\Delta k) = \sqrt{N} \]

By \( \sigma^2 = E((\Delta k)^2) = N \), it also holds that:

\[ b_2 = \frac{1}{NP(1 - P)} \]
\[ \sigma = E(\Delta k) = \sqrt{N} \]

QED
C.2 A Heuristic Finding in the Random Walk

We finally give an interesting result in the random walk. Even now, the research on random walk is under development. As we showed in Section 6.1.1.1, we can find self-similarity in the random walk. We moreover show a heuristic finding of the probability of long leads in a random walk. Consider a game in which a fair coin is tossed repeatedly. When the cumulative number of heads is greater than the cumulative number of tails, heads is in the lead. It retains that position until the cumulative number of tails is greater. The probability distribution indicates the probability of one side being in the lead for different percentages of the duration of the game. For example, if the coin is tossed 20 times, the probability that heads will be in the lead during the entire course of the game is 0.176, the same as the probability that it will never be in the lead. Surprisingly, the least likely situation is for the two sides to be in the lead equal amounts of time (Fig. C.2).

References


Fig. C.2 The probability of long leads
Canfield J, Hansen MV (2001) Chicken soup for the soul: 101 stories to open the heart and Rekindle the spirit (Chicken Soup for the Soul). Health Communications, Arlington
Thibaut JW, Kelley HH (1959) The social psychology of groups. Wiley, New York
# Name Index

<table>
<thead>
<tr>
<th>Name</th>
<th>Aoki, M., 196, 199, 200, 202</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Arrow, K.J., 14</td>
</tr>
<tr>
<td></td>
<td>Arthur, W.B., vii, 9, 12, 13, 65, 75, 106</td>
</tr>
<tr>
<td></td>
<td>Debreu, G., 14</td>
</tr>
<tr>
<td></td>
<td>Gale, D., 71</td>
</tr>
<tr>
<td></td>
<td>Galton, F., 165–167</td>
</tr>
<tr>
<td></td>
<td>Gossen, H.H., 8</td>
</tr>
<tr>
<td></td>
<td>Hayek, F.A., 121</td>
</tr>
<tr>
<td></td>
<td>Helbing, D., ix, 22–24</td>
</tr>
<tr>
<td></td>
<td>Holland, J.H., 66, 67, 80</td>
</tr>
<tr>
<td></td>
<td>Itokawa, H., 145, 159</td>
</tr>
<tr>
<td></td>
<td>Jevons, W.S., 8</td>
</tr>
<tr>
<td></td>
<td>Jones, D., 142</td>
</tr>
<tr>
<td></td>
<td>Leontief, W., 74</td>
</tr>
<tr>
<td></td>
<td>Mainzer, K., 9, 13, 27, 75, 135, 145, 166, 191</td>
</tr>
<tr>
<td></td>
<td>Malthus, T., 7</td>
</tr>
<tr>
<td></td>
<td>Manderbrot, B., 178</td>
</tr>
<tr>
<td></td>
<td>Marshall, A., 9, 10, 15</td>
</tr>
<tr>
<td></td>
<td>Maymin, P.Z., 182, 188, 190, 191</td>
</tr>
<tr>
<td></td>
<td>Menger, C., 8</td>
</tr>
<tr>
<td></td>
<td>Miura, B., viii, 22</td>
</tr>
<tr>
<td></td>
<td>Morishima, M., 78, 102</td>
</tr>
<tr>
<td></td>
<td>Potron, M., 102</td>
</tr>
<tr>
<td></td>
<td>Quetelet, L.A.J., 165</td>
</tr>
<tr>
<td></td>
<td>Ricardo, D., 7, 9</td>
</tr>
<tr>
<td></td>
<td>Robbins, L., 10–11</td>
</tr>
<tr>
<td></td>
<td>Schefold, B., viii, 71</td>
</tr>
<tr>
<td></td>
<td>Schumpeter, J.A., 145</td>
</tr>
<tr>
<td></td>
<td>Shibusawa, E., viii</td>
</tr>
<tr>
<td></td>
<td>Smith, A., viii–ix, 14, 22, 24</td>
</tr>
<tr>
<td></td>
<td>Sraffa, P., 9, 68–70, 74, 85, 121</td>
</tr>
<tr>
<td></td>
<td>Turing, A.M., 191</td>
</tr>
<tr>
<td></td>
<td>von Neumann, J., 7, 11, 12, 65</td>
</tr>
<tr>
<td></td>
<td>Wald, A., 46, 48</td>
</tr>
<tr>
<td></td>
<td>Walras, L., 74</td>
</tr>
<tr>
<td></td>
<td>Weidlich, H., 27, 30</td>
</tr>
</tbody>
</table>

© Springer Japan 2015
Subject Index

A
Action rules, 181, 183
Acyclic network, 81–84
Adaptive cruising system, vii
Arbitrage, 112
ARIMA, 60
Artificial intelligence, 144
Automaton, 179, 182, 186, 190–193
Auto-regressive model, 162–164
Avatamsaka game, 195–202
  payoff matrix, 201
  stochastic process, 200–202

B
Batch auction, 66, 107, 110, 114
Bearish state of mind, 178, 179, 183, 186
Bucket brigade algorithm, 66–68, 79
Bullish state of mind, 178, 179, 181, 183, 186

C
Central limit theorem (CLT), 140, 165, 207–213
Classical doctrines, viii
Classical matching (mechanism), 19, 104
Coincidence in human, 145
Complex adaptive system, 66
Complexities, 161–193
Complex price motions, 190
Confucius, viii, 22, 24, 25
Consumption, viii, 2–6, 35–37, 52–58, 61, 78, 120, 121
  pattern, 36, 52, 63
Contrarian, 179
Covariance analysis, 98
Creative coincidence, 13, 146–150
Credit assignment, 67
Cross-operation(al), 132–133, 135, 148, 150, 189
Cumulative distribution function (CDF), 168
Cycle length, 184, 191, 192
Cyclic network, 86, 87

D
Decreasing failure rate, 172
Degree centrality, 87
Degree of redundancy, 146
Demand function, 36, 48–49, 111
Demand law, 45–51
Density function, 41, 50, 169, 171, 212
DNA, 145, 191
Dow Jones index, 180–182
Dual instability, 87

E
Economic chivalry, 15
Economic science, vii–ix, 5, 153, 159
Efficient market hypothesis (EMH), 29, 161–165
Eigenvector centrality, 83, 86, 87, 91
English auctions, 109–110
Evolution, 1, 2, 7, 12, 13, 26, 30, 66, 67, 69, 91, 131–159
Evolving, 1–31, 53, 198
Ewens’ sampling formula, 156, 158, 199

© Springer Japan 2015
Y. Aruka, Evolutionary Foundations of Economic Science, Evolutionary Economics
and Social Complexity Science 1, DOI 10.1007/978-4-31-54844-7
Flash crashes, 135, 137–141, 144
Framing, 43, 44, 204
Futures markets, 107, 110–112, 119, 120, 122, 126, 142–143, 189, 199
Gaussian, 13, 122, 150, 163, 165–169, 177, 179
Generalized central limit theorem (GCLT), 168–170
General urn process, 155–158
Genetic algorithm, 11, 29, 66–74, 79–80
Giffen effects, 45, 46
Graph communities, 91, 93
Hayabusa’s return, 146–149
Hayek’s self-organization, 150
Heavy tail, 150, 170–178
Heterogeneity, 5, 7, 8, 21, 41, 43, 51
Heterogeneous interactions, 26, 27, 44, 196–197
High complexity, 145, 191
High-frequency trades(ing), 131–141, 143, 189
*Homo economicus*, ix, 21–25
Homogeneity, 8, 9
*Homo socialis*, ix, 21–25
Household consumption, 58
Impartial caring, 25, 26
Independence from irrelevant alternatives (IIA), 40
Independent, identically distributed (IID), 168
Information and communications, 27, 88–90
Inner mechanism, 20
Innovation, vii, 8, 10, 23, 65, 66, 75, 95, 111, 145–153, 199, 201
Instability, 139, 140
Inter-industrial network (analysis), 83, 86, 90, 91, 95–98
Intermediate goods, 8
Intermediate product, 2, 3
Internal rules, 179, 181–184
Inverse gamma distribution, 171
Invisible hands, vii, 10–17, 20, 21, 27, 29, 142
Invisible market, 132, 144
Ion engine, 148, 149
Itayose, 102, 105, 114, 117
Japan Aerospace Exploration Agency (JAXA), 146, 147
J-COM, 143, 144
Joint-production system, 71–74, 80–88
Learning, 36, 67
Leontief’s input–output model, 12
Lévy distribution, 168, 169, 171, 177
Limit order, 19, 119, 133, 140, 187, 189
Linear programming, 12, 85
List, 8
Logical depth, 142, 145, 146
Lognormal distribution, 172
Long-term capital management, 12
Look-back windows, 184, 186, 190–193
Loss-cut, 126
Low complexity, 145, 191
Machines, 29, 36, 111, 119, 190
Margin, 7, 15, 19, 143
Marginal revolution, 8
Market order, 19, 67, 102, 103, 109, 115, 133, 187, 189
Mark to the market, 123, 143
Matching mechanism, 18, 101–129
Mind in itself, 36, 45
Mind transition, 188
Minimum spanning trees, 80–81
Morality, viii, 22
Moving average (strategy), 60, 119, 162
Mozi, 25, 26
impartial caring, 25–27
school, 25, 26
Multinomial logit (MNL), 27, 37, 40
Mutual benefit, 26
Net foreign assets (NFA), 16
Network analysis, viii, 65–98, 151, 191
Nonlinear, 1, 9, 11, 12, 138
Nonlinear sciences, 138
Nonlinear system, 137
Non-self-averaging, 198, 200, 202
Normal distributions, 161, 166–168, 170, 212
<table>
<thead>
<tr>
<th>Subject Index</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>O</strong></td>
<td></td>
</tr>
<tr>
<td>Operations research, 11</td>
<td></td>
</tr>
<tr>
<td>Optimistic state of mind, 80, 179</td>
<td></td>
</tr>
<tr>
<td>Optimistic contrarian, 182</td>
<td></td>
</tr>
<tr>
<td>Ordinary life, 10–11, 13, 102</td>
<td></td>
</tr>
<tr>
<td>Osaka Dojima, 112</td>
<td></td>
</tr>
<tr>
<td>Other-regarding interests, 35–37</td>
<td></td>
</tr>
<tr>
<td><strong>P</strong></td>
<td></td>
</tr>
<tr>
<td>Pareto–Lévy distribution, 142, 177</td>
<td></td>
</tr>
<tr>
<td>Perron–Frobenius theorem, 85, 102</td>
<td></td>
</tr>
<tr>
<td>Pessimistic state of mind, 178, 179, 182, 185</td>
<td></td>
</tr>
<tr>
<td>Piemelal stipulation, 109</td>
<td></td>
</tr>
<tr>
<td>Pitman’s sampling formula, 200</td>
<td></td>
</tr>
<tr>
<td>Pitman’s Chinese restaurant process, 158–159</td>
<td></td>
</tr>
<tr>
<td>Poisson–Dirichlet distribution, 200</td>
<td></td>
</tr>
<tr>
<td>Poisson–Dirichlet model, 158, 159, 202</td>
<td></td>
</tr>
<tr>
<td>Polya distribution, 155</td>
<td></td>
</tr>
<tr>
<td>Polya urn process, 93, 151–154, 198–199</td>
<td></td>
</tr>
<tr>
<td>Polya urn rule, 152</td>
<td></td>
</tr>
<tr>
<td>Position management, 13, 112, 122, 126</td>
<td></td>
</tr>
<tr>
<td>Power law, viii, 13, 142, 152, 167, 170</td>
<td></td>
</tr>
<tr>
<td>Production system, 69, 70, 73, 74, 77, 80, 81, 85, 86, 88, 151</td>
<td></td>
</tr>
<tr>
<td><strong>R</strong></td>
<td></td>
</tr>
<tr>
<td>Random agents, 121–123, 125–129</td>
<td></td>
</tr>
<tr>
<td>Random matrix theory (RMT), 52, 53, 56, 61, 98</td>
<td></td>
</tr>
<tr>
<td>Random walk, 29, 140, 161, 165, 167, 170, 207–213</td>
<td></td>
</tr>
<tr>
<td>Recyclic network, 81</td>
<td></td>
</tr>
<tr>
<td>Re-domaining, vii, 131–141</td>
<td></td>
</tr>
<tr>
<td>Relaxed static stability, 144</td>
<td></td>
</tr>
<tr>
<td>Reproduction, 77</td>
<td></td>
</tr>
<tr>
<td>Reselling, 104, 105</td>
<td></td>
</tr>
<tr>
<td>Revolution, viii, 2, 5, 12, 18, 24</td>
<td></td>
</tr>
<tr>
<td>Rule discovery, 67</td>
<td></td>
</tr>
<tr>
<td><strong>S</strong></td>
<td></td>
</tr>
<tr>
<td>Self-regarding, 35–37</td>
<td></td>
</tr>
<tr>
<td>Self-similarity, 165, 167, 170</td>
<td></td>
</tr>
<tr>
<td>Service economy (dominance), 1–7</td>
<td></td>
</tr>
<tr>
<td>SF spread, 120–129</td>
<td></td>
</tr>
<tr>
<td>Shadow banking system (SBS), 142</td>
<td></td>
</tr>
<tr>
<td>Simplest trader dynamics, 186–187</td>
<td></td>
</tr>
<tr>
<td>Single autonomous engine, 178</td>
<td></td>
</tr>
<tr>
<td>Smart grid system, 29</td>
<td></td>
</tr>
<tr>
<td>Spot markets, 111, 116, 120, 125, 126</td>
<td></td>
</tr>
<tr>
<td>Spread, 21, 23, 51, 119</td>
<td></td>
</tr>
<tr>
<td>Sraffa joint-production system, 71–72</td>
<td></td>
</tr>
<tr>
<td>Sraffa system, 69</td>
<td></td>
</tr>
<tr>
<td>Sraffa theorem, 82</td>
<td></td>
</tr>
<tr>
<td>Stable distribution, 142, 166–170</td>
<td></td>
</tr>
<tr>
<td>Stage-setting moves, 67</td>
<td></td>
</tr>
<tr>
<td>Standard commodity, 9, 69–74</td>
<td></td>
</tr>
<tr>
<td><strong>T</strong></td>
<td></td>
</tr>
<tr>
<td>Tanimoto’s diagram, 197–199</td>
<td></td>
</tr>
<tr>
<td>Tâtonnement, 105</td>
<td></td>
</tr>
<tr>
<td>Taylor expansion, 209</td>
<td></td>
</tr>
<tr>
<td>Traders’ dynamics, 187</td>
<td></td>
</tr>
<tr>
<td>Traditional trader dynamics, 188</td>
<td></td>
</tr>
<tr>
<td>Transducer, 28, 179, 181, 185</td>
<td></td>
</tr>
<tr>
<td>Trend-follower, 178, 179</td>
<td></td>
</tr>
<tr>
<td><strong>U</strong></td>
<td></td>
</tr>
<tr>
<td>U-Mart Project, 111, 124</td>
<td></td>
</tr>
<tr>
<td>U-Mart system, 112, 116–119</td>
<td></td>
</tr>
<tr>
<td>Urn process, 146, 155–158</td>
<td></td>
</tr>
<tr>
<td><strong>V</strong></td>
<td></td>
</tr>
<tr>
<td>von Neumann solution, 68</td>
<td></td>
</tr>
<tr>
<td>von Neumann balanced growth model, 9</td>
<td></td>
</tr>
<tr>
<td>von Neumann balanced growth scheme of production, 3</td>
<td></td>
</tr>
<tr>
<td>von Neumann economic system, 67, 68</td>
<td></td>
</tr>
<tr>
<td>von Neumann economy, 77–79</td>
<td></td>
</tr>
<tr>
<td>von Neumann system, 67, 68, 71, 72</td>
<td></td>
</tr>
<tr>
<td><strong>Z</strong></td>
<td></td>
</tr>
<tr>
<td>Zaraba, 112, 114, 126</td>
<td></td>
</tr>
<tr>
<td>Zero-intelligence, 112, 116, 119</td>
<td></td>
</tr>
</tbody>
</table>